Positioning via Direct Localization in C-RAN Systems

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Abstract

Cloud Radio Access Network (C-RAN) is a prominent architecture for 5G wireless cellular system which is based on the centralization of baseband processing for multiple distributed radio units (RUs) at a control unit (CU). In this correspondence, it is proposed to leverage the C-RAN architecture to enable the implementation of direct, or one-step, localization of the position of mobile devices from the received signals at distributed RUs. With ideal connections between the CU and the RUs, direct localization is known to outperform the traditional indirect, or two-step, localization, whereby the source is located at a central node based on position-related measurements, such as time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA), or received signal strength (RSS), estimated at each RU separately. In a C-RAN system, however, this may not be the case due to the distortion caused by the quantization of the received signal at the RU needed to cope with the capacity limitation of the fronthaul links connecting RUs and CU. In this correspondence, the performance of direct localization is studied for a C-RAN system by accounting for the effect of fronthaul quantization. The analysis includes the derivation of Cramér-Rao Bound (CRB) on the squared position error (SPE) of direct localization with quantized observations, as well as the performance comparison of indirect localization and direct localization with or without dithering via numerical results.

Index Terms

Cloud Radio Access Network (C-RAN), localization, positioning, fronthaul, quantization, Cramér-Rao Bound (CRB).
I. INTRODUCTION

Localization based on existing wireless networks is currently an active research area due to its relevance in Global Positioning System (GPS)-denied environments, such as military applications, indoors localization, disaster response, emergency relief, surveillance and tactical systems [1]–[10]. The importance of the topic is attested by the recent Federal Communications Commission (FCC) specification for indoor positioning accuracy [11] and by the inclusion of various localization techniques in the Long Term Evolution (LTE) standard [12]. The traditional approach to source localization has been indirect, or two-step, localization. Accordingly, the distributed base stations (BSs) estimate position-related parameters, such as time of arrival (TOA) [1]–[5], time difference of arrival (TDOA) [2], [4], angle of arrival (AOA) [1], [4], [6]–[9] or received signal strength (RSS) [4], [10], which are transmitted to a control unit (CU) that determines the source’s position. For instance, in LTE, positioning is based on TDOA in the uplink, which is referred to as uplink TDOA (UTDOA) [12].

Cloud radio access network (C-RAN), a cloud-based network architecture, has emerged as one of leading technologies for 5G system and beyond [13]. In C-RANs, the BSs are connected to the CU by means of so called fronthaul links, as shown in Fig. 1. The fronthaul links carry sampled and quantized baseband signal to and from the CU. Thanks to the fronthaul links, the CU can carry out all baseband processing on behalf of the connected BSs. Since the BSs implement only radio functionalities, they are typically referred to as radio units (RUs). C-RANs hence enable the joint processing at the CU of the signals received by the RUs, making it possible to implement direct, or one-step, localization [1], [3], [6], [7], whereby the source’s position is determined directly from the received signals of the RUs.

With ideal fronthaul links, direct localization has been shown to outperform indirect localization [1], [6], [7]. However, in a C-RAN, due to the limited capacity of the fronthaul links, the baseband signals received by the RUs are quantized prior to transmission to the CU. In this work, we study the performance of direct localization by accounting for the distortion due to fronthaul quantization. In [1], [4], [6]–[8], localization in C-RAN systems was studied by assuming ideal compression by means of information theoretic
considerations, while here we consider a more practical scalar quantization that could be implemented in current state-of-the-art C-RAN systems. The analysis encompasses the derivation of the Cramér-Rao Bound (CRB) \[14\] on the squared position error (SPE) of direct localization based on quantized received signals, as well as numerical performance analysis of direct and indirect positioning schemes. A dithered quantization scheme \[15\] for direct positioning is also proposed that is seen to improve the performance.

In the following, after introducing the system and signal models, we shortly overview indirect localization in Section II. Direct localization without and with dithering is introduced for the C-RAN system in Section III and the CRB is derived in Section IV. Numerical examples and concluding remarks are presented in Section V and Section VI, respectively.

II. SYSTEM, SIGNAL MODELS AND PRELIMINARIES

In this section, we describe system and signal models, and review indirect localization \[1\], \[6\]–\[8\].

A. System Model

We consider the C-RAN system illustrated in Fig. 1 which consists of a single active source, \(N_r\) distributed RUs and a CU. The RUs may consist of different types of infrastructure nodes such as
macro/femto/pico base stations, relay stations or distributed antennas and the source may be a mobile
device. Each RU is equipped with an $M$-element antenna array, while the source is a single-antenna node.
The set of RUs, denoted as $\mathcal{N}_r = \{1, \ldots, N_r\}$, are located within a $D \times D$ square area, and all the
RUs are assumed to be synchronous. The source is located at a position $p = [x \ y]^T$, which is known a
priori to lie in a given region $A$, which is smaller than the overall square region $[3]$. Each RU $j \in \mathcal{N}_r$
is located at a position $p_j = [x_j \ y_j]^T$, and the positions of all RUs are assumed to be known to the
CU. The distance and angle between the source and the RU $j$ are defined as $d_j(p) = \|p - p_j\|$ and
$\phi_j(p) = \tan^{-1}((y - y_j)/(x - x_j))$, respectively. We assume the far-field conditions that the array aperture
is much smaller than the distance $d_j(p)$.

The RUs are connected to the CU via finite-capacity fronthaul links, and the CU aims at locating the
source based on quantized data transmitted from the RUs to the CU over the fronthaul links. Specifically,
each RU $j$ sends a message at a rate of $B_j$ bps/Hz to the CU, where the normalization is with respect
to the bandwidth of the signal transmitted by the source, as we will further discuss below. Note that the
fronthaul links can be either wireless, e.g., a microwave link, or wired, e.g., a coaxial cable or a fiber
optics link.

B. Signal Model

The source transmits a scaled version of a baseband waveform $s(t)$ with unit energy and single sided
bandwidth $W$ Hz at an unknown time $t_0$. The waveform $s(t)$ is known to the CU and it may represent
a training or synchronization signal. During the observation time $T$, the $M \times 1$ received signal vector at
the RU $j \in \mathcal{N}_r$ is hence expressed as

$$r_j(t) = b_j \alpha_j(p)s(t - \tau_j(p) - t_0) + z_j(t), \quad 0 \leq t \leq T, \quad (1)$$

where $b_j$ is a unknown complex scalar coefficient that describes the channel attenuation from the source
to the RU $j$; $\alpha_j(p)$ is the antenna array response at the RU $j$, which, assuming a uniform linear antenna
array (ULA) at each RU, is given as $\alpha_j(p) = 1/\sqrt{M}[1 \ e^{-i2\pi \Delta \cos \phi_j(p)/\lambda} \ldots e^{-i2\pi (M-1)\Delta \cos \phi_j(p)/\lambda}]^T$, with
the wavelength of the signal $\lambda$ and the antenna separation $\Delta$; $\tau_j(p)$ is the propagation delay of the path.
between the source and the RU $j$, which depends on the source’s position as $\tau_j(p) = d_j(p)/c$, with the propagation speed $c$; and $z_j(t)$ represents the contribution of noise and interference, and is assumed to be zero-mean white complex Gaussian with power $\sigma_j^2$. We assume that the source moves slowly enough and that the observation time $T$ is sufficiently long so that the inequalities $T \gg \max \tau_j(p) + t_0$ for $j \in \mathcal{N}_r$ are satisfied.

After sampling at the Nyquist rate, the discrete-time received signal (1) is given as

$$r_j[n] \triangleq r_j(nT_s) = b_j \alpha_j(p) s_j[n] + z_j[n], \quad 0 \leq n \leq N_s - 1, \quad (2)$$

where $T_s = 1/2W$ is the sampling period; $N_s = \lceil T/T_s \rceil$ is the number of time samples; and the sampled versions of the signal and the noise are denoted $s_j[n] \triangleq s(nT_s - \tau_j(p) - t_0)$ and $z_j[n] \triangleq z_j(nT_s)$, respectively. Taking the discrete Fourier transform (DFT) of the discrete signal (2), we get the received signal in the frequency domain as

$$R_j(k) = b_j \alpha_j(p) S(k) e^{-iw_k(\tau_j(p) + t_0)} + Z_j(k), \quad 0 \leq k \leq N_s - 1, \quad (3)$$

where $w_k = 2\pi k/(N_sT_s)$; and $R_j(k)$, $S(k)$ and $Z_j(k)$ indicate the DFT coefficients of the respective time-domain signals $r_j[n]$, $s(nT_s)$, and $z_j[n]$. Given the assumptions above, we have the equality $\sum_{k=0}^{N_s-1} |S(k)|^2 = 1$, and the DFT coefficients of the noise $Z_j(k)$ are uncorrelated and with power $\sigma_j^2$, for $0 \leq k \leq N_s - 1$. We observe that, in (3), the information about the source’s location is embedded in two ways in the received signal, namely in time delay $\tau_j(p)$ and in the array response $\alpha_j(p)$.

The RUs quantize and forward the received signals in (3) to the CU for positioning under the fronthaul constraint. As a practical solution, prescribed, e.g., by the CPRI standard [16], we assume that each RU applies the same uniform quantizer for all frequencies and antennas. To evaluate the localization accuracy, we adopt the SPE [2], [3], [14]

$$\rho = \mathbb{E} [\| \hat{p} - p \|^2], \quad (4)$$

as the performance criterion of interest, where $\hat{p}$ is the estimated source’s position.
C. Indirect Localization

In the conventional indirect localization method, each RU $j$ estimates the parameters $\tau_j(p)$ and $\phi_j(p)$, which are referred to as TOA and AOA measurements, respectively; and then these estimates are sent to the CU, which locates the source by maximum likelihood based on the forwarded estimates. The estimates are obtained here by means of the multiple signal classification (MUSIC) algorithm [5], [9], as in [6], but other super-resolution techniques are also possible. Note that, under fronthaul capacity constraints, the AOA and TOA measurements at RU $j$ should be quantized prior to transmission to the CU. Nevertheless, assuming that the number of samples $N_s$ is large enough, the number of bits $N_sB_j$ available for quantization of the estimates is large enough to make the corresponding distortion due to quantization negligible.

III. DIRECT LOCALIZATION

With direct localization, the position of the source is estimated directly from the received signals at all the RUs [1], [3], [6], [7]. Due to the fronthaul capacity limitation of the C-RAN system, we consider that each RU $j$ sends the quantized DFT coefficients to the CU over limited fronthaul links with $B_j$ bits per sample. We adopt standard uniform scalar quantization, with $B_j/2M$ bits, or equivalently $L_j = 2^{B_j/2M}$ quantization levels, for either the real or imaginary parts of the signal received on any of the $M$ received antennas. The dynamic range $[-R_{j\text{max}}, R_{j\text{max}}]$ of the uniform quantizer applied at each RU $j$ to all antennas is fixed based on preliminary Monte Carlo experiments aimed at guaranteeing that the peak-to-peak interval includes the received signals with high probability when considering a given random distribution of the source, and of the channel coefficients, as well as the distribution of the noise (see also Section V).

We consider two different forms of uniform scalar quantization. In the conventional one, each sample $R_j(k)$ is quantized to the closest value $\hat{R}_j(k)$ in the set of representation points $\{-R_{j\text{max}} + (l - 1)\Delta_j\}$ for $l = 1, \ldots, L_j$, where $\Delta_j = 2R_{j\text{max}}/(L_j - 1)$ is the step size of the quantizer. We also study dithered quantization [15], whereby the signal to be quantized is $R_j(k) + D_j(k)$, where $D_j(k)$ is a dither signal, independent of the received signal $R_j(k)$ and known to the CU. The addition of a dither signal is known
to potentially improve the accuracy of a quantized signal \[15\]. Here, we take the dithers \(D_j(k)\) to be independent and uniformly distributed in the interval \([-\Delta_j/I_j, \Delta_j/I_j]\), where \(I_j\) is a parameter that can be chosen to minimize the SPE \[4\].

Based on the quantized signals \(\{\hat{R}_j(k)\}\), obtained from the received signal with or without dithering, the CU estimates the source’s position. The estimator under consideration is the same as in \[1\], \[6\], \[7\], except that it is applied to the quantized signals \(\hat{R}_j(k)\), rather than to the received signals \(R_j(k)\) due to the limited fronthaul capacity. Specifically, following \[1\], \[6\], \[7\], the estimate is the minimizer of the cost function \(\sum_{j=1}^{N_r} C_j(b_j, t_0, p)\), with

\[
C_j(b_j, t_0, p) = \begin{cases} \sum_{k=0}^{N_s-1} \left\| \hat{R}_j(k) - b_j \alpha_j(p) S(k) e^{-i\omega_k(\tau_j(p)+t_0)} \right\|^2, & \text{w/o dithering,} \\ \sum_{k=0}^{N_s-1} \left\| \hat{R}_j(k) - D_j(k) - b_j \alpha_j(p) S(k) e^{-i\omega_k(\tau_j(p)+t_0)} \right\|^2, & \text{w/ dithering,} \end{cases}
\]

where, in the case of dithering, the dither is subtracted at the CU (see, e.g., \[15\]). To solve the minimization, the unknown parameter \(b_j\) is first obtained by minimizing the cost function \(\sum_{j=1}^{N_r} C_j(b_j, t_0, p)\) as

\[
b_j^* (p, t_0) = \arg\min_{b_j} \sum_{j=1}^{N_r} C_j(b_j, t_0, p) = \begin{cases} [S_j \otimes \alpha_j(p)]^H \hat{R}_j, & \text{w/o dithering,} \\ [S_j \otimes \alpha_j(p)]^H (\hat{R}_j - D_j), & \text{w/ dithering,} \end{cases}
\]

where \(S_j = [S(0)e^{-i\omega_0(\tau_j(p)+t_0)} \cdots S(N_s-1)e^{-i\omega(N_s-1)(\tau_j(p)+t_0)}]^T\) and \(\hat{R}_j = [\hat{R}_j^T(0) \cdots \hat{R}_j^T(N_s-1)]^T\). To deal with the unknown parameter \(t_0\), we proceed as follows \[1\]. For any candidate for the source’s position \(p\) within \(A\), we define the matrix \(V(p)\) as \(V(p) = S^H U(p)\), where \(S = \text{diag}\{S(0), \ldots, S(N_s-1)\}\) and \(U(p) = [u_1(p) \cdots u_{N_s}(p)]\), where \(u_j(p) = [e^{i\omega(\tau_j(p))} \alpha_j^H(p) \hat{R}_j(0) \cdots e^{i\omega(N_s-1)(\tau_j(p))} \alpha_j^H(p) \hat{R}_j(N_s-1)]^T\) if no dithering is used, or \(u_j(p) = [e^{i\omega(\tau_j(p))} \alpha_j^H(p) (\hat{R}_j(0) - D_j(0)) \cdots e^{i\omega(N_s-1)(\tau_j(p))} \alpha_j^H(p) (\hat{R}_j(N_s-1) - D_j(N_s-1))]^T\) if dithering is used. Then, we fix a positive integer \(q_{t_0}\) that defines the resolution of the estimate of \(t_0\) \[1\]. Denote as \([a]_k\) the \(k\)th element of vector \(a\) and as \([A]_{k,n}\) the \((k, n)\)th element of matrix \(A\). The minimization over \(t_0\) and over \(p\) can be jointly performed as maximizing the maximum element of \(v(p)\) as

\[
p^* = \arg\max_p \{\max_{1 \leq k \leq q_{t_0} N_s} [v(p)]_k\}\), where \(v(p) = \sum_{j=1}^{N_r} |FV_j(p)|^2\) with \(V_j(p)\) being the \(j\)th column of \(V(p)\) and \(F\) being the \(q_{t_0} N_s\)-point FFT matrix whose components are \([F]_{k,n} = e^{-i2\pi(k-1)(n-1)/(q_{t_0} N_s)}\) for \(1 \leq k, n \leq q_{t_0} N_s\) \[1\]. Note that, if \(q_{t_0} \geq 2\), \(V_j(p)\) is padded with trailing zeros to length \(q_{t_0} N_s\).
IV. CRB FOR DIRECT LOCALIZATION

In this section, we derive a lower bound on the SPE (4) for the direct method without dithering via the CRB \[14\]

\[
\rho \geq \text{tr} \left\{ J^{-1}(p) \right\},
\]

(7)

where \( J(p) \) is Equivalent Fisher Information Matrix (EFIM) \[2\], \[3\] for the estimation of the source’s position \( p \).

To elaborate, we denote the uniform quantization function as \( Q_j(x) = \{l|q_{j,l-1}(\Delta_j) < x \leq q_{j,l}(\Delta_j); \ l = \{1, \ldots, L_j\}\} \), where \( q_{j,l}(\Delta_j) = -R_j^{\max} + (l - 0.5)\Delta_j \) is the \( l \)th quantization threshold for the RU \( j \) with \( q_{j,0} = -\infty \) and \( q_{j,L_j} = \infty \). Recall that the signal \( S = \{S(k)\} \) is known to the CU and that the transmit time \( t_0 \) is cancelled out by direct localization, and hence they are treated as known parameters \[6\]. The probability of the quantized signals available at the CU for given unknown parameter vector \( \theta = [p^T \ b_1^T \cdots b_{N_r}^T]^T \) with \( b_j = [b_j^R \ b_j^I]^T \) is then given as

\[
P(\hat{R}; \theta) = P_j \left( \prod_{j=1}^{N_r} \prod_{k=0}^{N_s-1} \prod_{m=1}^{M} \prod_{l=1}^{L_j} P_{j,l}(Q_j([R_j^\zeta(k)]_m); \theta_j) \delta(Q_j([R_j^\zeta(k)]_m) - l), \right)
\]

(8)

where \( \delta(\cdot) \) is the Kronecker-delta function, i.e., \( \delta(0) = 1 \) and \( \delta(x) = 0 \) for \( x \neq 0 \); and \( P_{j,l}(Q_j([R_j^\zeta(k)]_m); \theta_j) \) is the probability that either the real or imaginary part of the received signal, \([R_j^R(k)]_m\) or \([R_j^I(k)]_m\), for the \( m \)th antenna at the RU \( j \) takes on a specific value \( l \) among the \( L_j \) quantization levels, i.e., 

\[
P_{j,l}(Q_j([R_j^\zeta(k)]_m); \theta_j) = \Phi \left( \frac{q_{j,l}(\Delta_j) - f_{j,k,m}^\zeta(\theta_j)}{\sigma_j/\sqrt{2}} \right) - \Phi \left( \frac{q_{j,l-1}(\Delta_j) - f_{j,k,m}^\zeta(\theta_j)}{\sigma_j/\sqrt{2}} \right),
\]

(9)

with \( \zeta \in \{\Re, \Im\} \) and \( \theta_j = [p^T \ b_j^T]^T \). In \[9\], \( \Phi(\cdot) \) is the complementary cumulative distribution function of the standard Gaussian distribution \( \Phi(x) = \int_{-\infty}^{x} e^{-t^2/2}/\sqrt{2\pi}dt \), and we have defined the noiseless received signal as \( f_{j,k,m}(\theta_j) = b_j[\alpha_j(p)]_m S(k) e^{-i\omega_k(\theta_j) + \tau_0} \).

The log-likelihood function is given by \( L(\theta) = \ln P(\hat{R}; \theta) \), and the FIM \( J(\theta) \) for the unknown parameter vector \( \theta \) can be written as \( J(\theta) = -E_{\hat{R}}[\nabla_\theta \nabla_\theta^T L(\theta)] \[14\]. The CRB in \[7\] can be then obtained from the EFIM for the location \( p \), which is given as

\[
J(p) = X - YZ^{-1}Y^T,
\]

(10)
where \( X = [J(\theta)]_{(1:2,1:2)} \); \( Y = [J(\theta)]_{(1:2,3:2N_r+2)} \); and \( Z = [J(\theta)]_{(3:2N_r+2,3:2N_r+2)} \), with \([A]_{(a:b,c:d)}\) being the sub-matrix of \( A \) corresponding to the \( a \)th to the \( b \)th rows and from the \( c \)th to the \( d \)th columns. In Appendix A, we calculate these matrices as

\[
X = \sum_{j=1}^{N_r} U_j \Psi_j U_j^T; \quad Y = [U_1, \Psi_1] V^T \cdots U_{N_r}, \Psi_{N_r}] V^T]; \quad \text{and} \quad Z = \text{diag}\{V \Psi_1 V^T, \ldots, V \Psi_{N_r} V^T\},
\]

where \( V = [0_2, 0_2, I_2] \); \( U_j = [\cos \phi_j(p)/c - \sin \phi_j(p)/d_j(p) \ 0_2^T] \); \( \sin \phi_j(p)/c \cos \phi_j(p)/d_j(p) \ 0_2^T \); and

\[
[\Psi_j]_{p,q} = \sum_{j,k,m,\zeta,l} \frac{(\Gamma_{j,k,m,l} - \Gamma_{j,k,m,l-1})^2}{\pi \sigma_j^2 P_j(Q_j(\Psi_j^R(k); \zeta_j))} \frac{\nabla_{\theta_j} f_{j,k,m}(\theta_j) \nabla_{\theta_j} f_{j,k,m}(\theta_j)}{\lambda_j^2},
\]

for \( 1 \leq p, q \leq 4 \), with \( \theta_j = [r_j(p) \ p_j(p) \ b_j^T]^T \); \( \Gamma_{j,k,m,l} = e^{-(q_j,\Delta_j-f_j^{0}_{j,k,m}(\theta_j))^2/\sigma_j^2} \); and the required derivatives \( \nabla_{\theta_j} f_{j,k,m}^R(\theta_j) = \Re\{\cdots \} \); \( \nabla_{\theta_j} f_{j,k,m}^3(\theta_j) = \Im\{\cdots \} \); \( \nabla_{\phi_j} f_{j,k,m}^R(\theta_j) = \Re\{\cdots \} \); \( \nabla_{\phi_j} f_{j,k,m}^3(\theta_j) = \Im\{\cdots \} \); \( \nabla_{b_j} f_{j,k,m}^R(\theta_j) = \Re\{\cdots \} \); \( \nabla_{b_j} f_{j,k,m}^3(\theta_j) = \Im\{\cdots \} \); \( \nabla_{b_j} f_{j,k,m}^R(\theta_j) = \Re\{\cdots \} \); \( \nabla_{b_j} f_{j,k,m}^3(\theta_j) = \Im\{\cdots \} \).}

V. Numerical Results

In this section, we compare the performance of direct and indirect localization in the C-RAN system. We consider a network with \( N_r = 4 \) RUs placed at the vertices of a square area with side of length \( D = 4 \) km, while the source is uniformly and randomly distributed within a \( 3 \times 3 \) km² area of \( A \) centered in the entire region. Each RU is assumed to be equipped with an \( M = 8 \) element ULA with antenna separation \( \Delta = \lambda/2 \). The sampled waveform is \( s(n) = \text{sinc}(t/T_s)/\sqrt{N_s} \text{e}^{-i\omega_k (r_j(p+t_0))} \); with bandwidth \( W = 1/(2T_s) = 1 \) kHz so that \( T_s = 0.5 \) ms, and we have \( N_s = 8 \). We assume that the channel coefficients \( b_j \) are independent and Rician with Rician factor \( K = 20 \) dB. Also, we impose an equal fronthaul capacity constraint \( B_j = B \), and assume an equal noise variance \( \sigma_j^2 = 1 \) for the RU \( j \in N_r \). We set \( q_{t_0} = 1 \) in the direct localization scheme for estimating \( t_0 \). We evaluate the root mean squared (RMS) error as performance metric defined as

\[ \text{RMS error} = \sqrt{\frac{\sum_{n=1}^{N} \rho_n}{N}}, \quad \text{where} \quad \rho_n = ||\hat{p}_n - p||^2 \quad \text{with} \quad N \quad \text{being the number of experiments and} \quad \hat{p}_n \quad \text{being the estimated source location of the} \quad n \text{th experiments}. \]

For reference, we consider the performance of direct localization with ideal fronthaul links and the CRB.
Fig. 2. RMS error versus fronthaul capacity $B/M$ bps/Hz/antenna for SNR = 0 dB ($N_r = 4$, $M = 8$, $N_s = 8$ and $T_s = 0.5$ ms).

Fig. 3. RMS error versus fronthaul capacity $B/M$ bps/Hz/antenna for SNR = 5 dB ($N_r = 4$, $M = 8$, $N_s = 8$ and $T_s = 0.5$ ms).

Fig. 2 and Fig. 3 show the RMS error versus the fronthaul capacity constraint $B/M$ (bps/Hz/antenna) for SNR = 0 dB and SNR = 5 dB, respectively. First, we observe that, when the SNR is sufficiently large, as in Fig. 3 (SNR = 5 dB), indirect localization can outperform direct localization if the fronthaul capacity is small enough. This is due to the distortion caused by fronthaul quantization in the presence of a small resolution. Note that this is not the case at lower SNR, here SNR = 0 dB, given that, in this
regime, the signal degradation due to the additive noise overwhelms the loss due to quantization. As long as the fronthaul capacity is large enough, as expected, direct localization has the potential to significantly outperform indirect localization with additional marginal gains achievable via dithering. The gains of dithering are more relevant at higher SNR, in which regime the effect of dithering is not masked by the noise. For instance, the gain of dithering increases from 5.9% at SNR = 0 dB to 8.7% at SNR = 5 dB in terms of RMS with $B/M = 2$. Furthermore, the CRB, as well as the performance with ideal fronthaul, are approached by the direct scheme as the fronthaul capacity increases.

VI. CONCLUDING REMARKS

The C-RAN architecture, with its centralized baseband processing at a control unit (CU), enables the implementation of direct, or one-step, localization. While direct localization outperforms the traditional indirect, or two-step, localization in the presence of ideal connections between the CU and the radio units (RUs), this may not be the case in the C-RAN due to the distortion caused by quantization on the fronthaul links. In this correspondence, we have studied the performance of direct and indirect localization in the C-RAN by means of analysis, which includes the calculation of the CRB, and via numerical results. Interesting open problems concern the study of the impact of imperfect synchronization among the RUs.

APPENDIX A

CALCULATION OF EFIM FOR DIRECT LOCALIZATION

In this appendix, we calculate the EFIM $J(p)$ of direct localization in (10). Similar to [2], [3], since the source is localizable, the mapping of $\theta = [p^T b_1^T \cdots b_{N_r}^T]^T$ to $\tilde{\theta} = [\tilde{\theta}_1^T \cdots \tilde{\theta}_{N_r}^T]^T$ with $\tilde{\theta}_j = [\tau_j(p) \phi_j(p) b_j^T]^T$ is a bijection. Then, we can have the relationship $J(\theta) = TJ(\tilde{\theta})T^T$, where $T = \partial \tilde{\theta} / \partial \theta = [U_1 \cdots U_{N_r}; V_1 \cdots V_{N_r}]$ is the Jacobian matrix; and $J(\tilde{\theta}) = -E_R[\nabla_{\tilde{\theta}} \nabla_{\tilde{\theta}}^T L(\theta)]$, with $U_j$ defined in (10) and $V_j \in \mathbb{R}^{2N_r \times 4}$ having all zeros except for $[V_j]_{2(j-1)+1:2j, 3:4} = I_2$. Referring [10], $J(\tilde{\theta})$ can be calculated as $J(\tilde{\theta}) = \text{diag}\{\Psi_1, \ldots, \Psi_{N_r}\}$, and using the relationship $J(\theta) = TJ(\tilde{\theta})T^T$, $J(\theta)$ can be written as
\[ J(\theta) = [X \ Y; Y^T \ Z], \] where \( X, Y \) and \( Z \) are given in (10). Finally, by applying the Schur complement (see, e.g., [17]), we can have the EFIM \( J(p) \) in (10).

**REFERENCES**


