A Computationally Efficient Fano-Based Sequential Detection Algorithm for V-BLAST Systems

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SUMMARY We present a computationally efficient sequential detection scheme using a modified Fano algorithm (MFA) for V-BLAST systems. The proposed algorithm consists of the following three steps: initialization, tree searching, and optimal selection. In the first step, the proposed detection scheme chooses several candidate symbols at the tree level of one. Based on these symbols, the MFA then finds the remaining transmitted symbols from the second tree level in the original tree structure. Finally, an optimal symbol sequence is decided among the most likely candidate sequences searched in the previous step. Computer simulation shows that the proposed scheme yields significant saving in complexity with very small performance degradation compared with that of sphere detection (SD). The maximum likelihood (ML) detection is proved to provide better performance than any other V-BLAST detectors, but it is generally difficult to be used in practical systems due to its extreme complexity. When any other V-BLAST detectors, but it is generally difficult to be used in practical systems due to its extreme complexity especially when many transmit antennas together with higher modulation are involved. As possible solutions to this problem, there have been many contributions achieving near-ML performance with low complexity such as a sphere detection (SD) [2], [3]. Similarly, a modified Fano detection (MFD) scheme with an additional backward tracing has been introduced in [4]. The detailed procedures of MFD and a brief comparison with other similar approaches, metric guided (MG) algorithm applying basic steps of Fano algorithm, were discussed therein. It was shown that the MFD has much less computational complexity with comparable performance to ML. However, it still requires the unnecessary complexity caused by repeated searching of the same candidate sequence.

Motivated by this, we propose a new Fano-based sequential detection (FBSD) algorithm for V-BLAST systems. The proposed algorithm is organized with three interrelated procedures: (i) initialization, (ii) tree searching, and (iii) optimal selection. The FBSD first chooses as many as desired candidate symbols at the first level of a M-ary tree structure, depending on the channel condition. Next, a modified Fano algorithm (MFA) finds the candidate sequences for each candidate symbol. Finally, an optimal sequence is selected among the candidate sequences. Not only improving the system performance but also reducing the receiver complexity, while the proposed FBSD utilizes the candidate symbol-based tree searching, the MFD uses the additional backward tracing based tree searching. Simulation results demonstrate that the FBSD yields the average bit error rate (BER) performance close to that of SD with significant reduction in complexity.

Notation: Throughout this letter, bold symbols denote matrices or vectors. \( (\cdot)^T \) and \( (\cdot)^H \) denote transpose and Hermitian transpose. \( \mathbf{C}^N \) and \( |\cdot| \) denote the set of all complex \( M \times 1 \) vectors and the Euclidean distance. \( \mathbf{I}_n \) and \( \mathbf{0}_{n,1} \) represent the \( n \times n \) identity matrix and the \( n \times 1 \) zero vector. \( L_i \) denotes the average number of real operations computed at the \( i \)-th step for SD and at the \( i \)-th tree level for MFD and FBSD.

1. Introduction

In wireless communications, the V-BLAST (Vertical Bell Labs Layered Space-Time) system has been considered as an efficient architecture for realizing very high data rates when the channel exhibits rich scattering [1]. The maximum likelihood (ML) detection is proved to provide better performance than any other V-BLAST detectors, but it is generally difficult to be used in practical systems due to its extreme complexity especially when many transmit antennas together with higher modulation are involved. As possible solutions to this problem, there have been many contributions achieving near-ML performance with low complexity such as a sphere detection (SD) [2], [3]. Similarly, a modified Fano detection (MFD) scheme with an additional backward tracing has been introduced in [4]. The detailed procedures of MFD and a brief comparison with other similar approaches, metric guided (MG) algorithm applying basic steps of Fano algorithm, were discussed therein. It was shown that the MFD has much less computational complexity with comparable performance to ML. However, it still requires the unnecessary complexity caused by repeated searching of the same candidate sequence.

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2. System Description

We consider a spatial multiplexing MIMO system with \( n_t \) transmit and \( n_r \) (\( \geq n_t \)) receive antennas. Assuming ideal timing and symbol-synchronous receiver, the complex base-band equivalent model of the received signal vector can be described as

\[ \mathbf{y} = \mathbf{H} \mathbf{s} + \mathbf{n} \]

where \( \mathbf{s} \in \mathbf{C}^{n_t} \) denotes the vector of transmitted symbols from \( M\)-PSK or \( M\)-QAM, \( \mathbf{y} \in \mathbf{C}^{n_r} \) denotes the received signal vector, \( \mathbf{n} \in \mathbf{C}^{n_r} \) denotes the zero-mean complex additive white Gaussian noise with variance \( \sigma_n^2 \), and the \( n_r \times n_t \) channel matrix \( \mathbf{H} \) contains uncorrelated complex Gaussian fading coefficients with unit variance, which is assumed to be known perfectly at the receiver.

3. Proposed V-BLAST Detection Algorithm

Figure 1 presents the flow chart of the FBSD with three interrelated steps. In the following subsections, we describe the detailed procedures of each step.

3.1 Initialization

In this step, two different tasks are performed to initiate the candidate symbol-based tree searching. One is that \( M \)-ary
Tree structure is constructed using QR decomposition of the channel matrix. Unless otherwise stated, we assume that each tree level corresponds to the detection order $n_1, \cdots, 1$ due to the structure of $R$. The other is that the number of candidate symbols is decided at the first tree level.

To diminish the error propagation and the noise enhancement simultaneously, we employ minimum mean square error-sorted QR decomposition (MMSE-SQRD) [5] of the extended channel matrix

$$H = \begin{bmatrix} H & \sigma_n I_n \end{bmatrix} = QR = \begin{bmatrix} Q & Q_1 \end{bmatrix} R,$$

where $(n_r+n_t) \times n_t$ unitary matrix $Q$ is partitioned into the $n_r \times n_t$ matrix $Q$ and the $n_t \times n_t$ matrix $Q_1$, and $R$ is the $n_t \times n_t$ upper triangular matrix. Note that the column of $H$ is sorted in order of maximum norm prior to each orthogonalization step while it is reordered in order of minimum norm in [5]. This idea is motivated by the observation that more reliable signal detection can be achieved with the aid of candidate symbol at lower than higher level of tree. Multiplying the extended received signal $y = [y \; 0_{n_t}]^T$ with $Q^T$ yields

$$Q^T y = \begin{bmatrix} R s + w, & 0_{n_r,1} \end{bmatrix}.$$  

From Eq. (3), a filtered received vector can thus be obtained as

$$\bar{y} = R s + w,$$

where $w = Q^T n$, the statistical properties of which remain unchanged due to the $n_r \times n_t$ matrix $Q$. Using the upper triangular structure of $R$, the $k$-th component of $\bar{y}$ is given by

$$\bar{y}_k = r_{k,k} s_k + \sum_{i=k+1}^{n_t} r_{k,i} s_i + w_k,$$

where $r_{k,k}$ represents the $(k, i)$-th element of $R$.

Next, we consider how to select the number of candidate symbols called herein $N_{cs}$. Here, the $N_{cs}$ candidate symbols are chosen by computing the Euclidean distance between the estimated signal $\hat{s}_n$ and the constellation points and by sorting them with minimum Euclidean distance. Since the signal to noise ratio (SNR) at each tree level can be determined from Eq. (5) assuming correct previous decisions, it is straightforward that any small diagonal term of $R$ results in signal distortion. Among them, $r_{n,n}$ required for detecting the first signal may have a critical affect on the system performance in recursive detection because of the risk of error propagation. This intuition provides the idea of deciding $N_{cs}$. That is, if $N_{cs}$ is suitably chosen based on the range of $r_{n,n}$, we obtain a reasonable tradeoff between the performance and the corresponding complexity. For a simple example, $N_{cs}$ can be assigned by computer search to get the desired performance and complexity when 16-QAM is used as follows

$$N_{cs} = \begin{cases} 1 & \text{if } r_{n,n} > 1 \\ 12 & \text{if } 0.5 < r_{n,n} \leq 1 \\ 16 & \text{otherwise} \end{cases}.$$  

We see from Eq. (6) that $N_{cs}$ is assigned to be larger as $r_{n,n}$ has a smaller value because the smaller value of $r_{n,n}$ causes the error probability of the first detected signal to increase.

### 3.2 Tree Searching

In this step, the FBSD searches the most likely candidate sequences using the MFA, where the candidate sequences include each candidate symbol obtained from the first step. More specifically, let the $N_{cs}$ estimated candidate symbols at the tree level of one be $\hat{s}^{1}_{n}, \cdots, \hat{s}^{N_{cs}}_{n}$. Based on these symbols, the MFA finds the remaining transmitted symbols from the tree depth of two in the original tree structure. In particular, it exploits the partial branches, to avoid an exhaustive tree search, instead of full branches unlike the MFD when the tree structure extends. We denote the number of partial branches as $N_b$.

Since the MFA follows the same procedures as the MFD except only for the additional backward tracing, we also employ the Fano-like metric bias introduced in [4] and compute the metric for the $i$-th branch at the tree depth of $k$ assuming the $j$-th candidate symbol $\hat{s}^{j}_{n}$ as

$$B^{i}_{i,k} = \left[ \frac{x^{i}_{j}}{r_{k,k}} - c_{i} \right]^2 + F_{k-1},$$

$$x_{k}^{j} \pm \bar{y}_k - r_{n,n} \hat{s}^{j}_{n} - \sum_{j=1}^{n_t} r_{k,j} s_j,$$

$$k = 2, \cdots, n_t, \; j = 1, \cdots, N_{cs}, \; i = 1, \cdots, N_b,$$

where $c_{i}$ and $F_{k-1}$ denote the $i$-th constellation point and the Fano-like metric bias at the $(k - 1)$-th depth of tree respectively. Using the branch metrics calculated in Eq. (7), the MFA operates until it finds the desired number of the candidate sequences.

### 3.3 Optimal Selection

As a final step of the proposed algorithm, the FBSD calculates the following metric for each candidate sequence

$$N_{cs} = \begin{cases} 1 & \text{if } r_{n,n} > 1 \\ 12 & \text{if } 0.5 < r_{n,n} \leq 1 \\ 16 & \text{otherwise} \end{cases}.$$
In this letter, we have proposed a new Fano-based sequential detection (FBSD) technique for V-BLAST systems. Simulation results illustrate that the FBSD provides an effective way to reduce the receiver complexity while it yields the near-SD performance. Therefore, we conclude that the proposed FBSD can be one of the possible solutions to the complexity problem in practical V-BLAST systems.

5. Conclusions

For complexity comparison, Table 1 provides the evaluated complexity for each detection algorithm based on the same approach in [4]. Here, note that for MMSE-OSIC, we consider the matrix pseudoinverse with the modified Gramm-Schmidt QR decomposition instead of Householder QR decomposition used in [4] for fair complexity comparison. In addition, Fig. 3 is plotted using the complexity analysis presented in Table 1. It is seen that the average complexity of the FBSD is considerably less than those of the SD and the MMSE-OSIC at low SNR region while it is comparable to that of the MMSE-OSIC at high SNR region. In particular, the FBSD remarkably reduce the receiver complexity with the same performance, compared to the MFD with \( N_{bm} = 400 \) and \( N_{m_{mm}} = 4 \). This is because \( N_{cs} \) with a maximum value (16 in our case) is limited to avoid exhaustive search especially at low SNR region.

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### References


