Achievable Rates for Transmitter Cooperation in wireless networks:

Pure Relay and Time Division Successive Broadcasting

Keonkook Lee, Na Young Kim, Namjeong Lee, and Joonhyuk Kang

School of Engineering (ECE), Information and Communications University (ICU)

{rjsmrnl, nykim, yell06, jhkang} @icu.ac.kr

ABSTRACT

In this paper, we investigate an achievable rate for two transmitter cooperation schemes, Pure Relay (PR) and Time Division Successive Broadcasting (TDSB). For these two strategies, we expand the existing 1-dimensional analysis into 2-dimensional space in wireless networks. We show that cooperation provides capacity improvement over noncooperation model. In addition, possible problems and their solutions are addressed.

1. INTRODUCTION

The mobile communication market is growing very rapidly. As a result, new technology to improve spectral efficiency including multiple input multiple output (MIMO) is researched. Yet, MIMO system needs multiple antennas in a mobile device, which has hardware restriction. Moreover it also may have user’s inconvenience, too. To resolve this problem, cooperative communication has been proposed recently. It is also called cooperation MIMO relaying (CMIMOR) or virtual antenna array (VAA). Especially, cooperative communication schemes have been researched much in Ad-hoc, since Ad-hoc networks consist of many number of spread nodes to communicate with other nodes without help of base station (BS). Among many introduced cooperation schemes, we apply Pure Relay (PR) and Time Division Successive Broadcasting (TDSB) models to wireless networks and for these two strategies, expand them into 2-dimensional space. Then, problems that may happen and their solutions are considered.

The paper is organized as follows. Section 2 shows a common system model, which is used in this paper. Achievable rate of PR and TDSB is investigated in section 3. Finally, we conclude the paper in section 4.

2. SYSTEM MODEL

Figure 1 represents the system model used in this paper. There are three nodes to participate the communication. Source node (S) sends a message to the Destination node (D) and relay station (R) may help the transmission. In this scenario, source and destination which have square shape are fixed. Relay station shaped by ellipse is assumed to move in given range, which is depicted in Figure 2. Moving relay station means that there may be many relay station in given ranges, which are chosen as a relay by source. The variable position of relay station is restricted to $r_1$ and $\theta$, where $0 \leq r_1 \leq 1$, $-\pi/4 \leq \theta \leq \pi/4$ respectively. This range is based on relay selection, which can increase the achievable rate. If $r_1$ and $\theta$ are determined, consequently $r_2$ is decided. The distance between source and destination is normalized to 1, for convenience.

The system assumes an AWGN environment. Suppose that $x_{ab}$ denotes the transmitted signal from ‘a’ to ‘b’. Then there are three transmitted signals, $x_{sr}$, $x_{rd}$, and $x_{sd}$. Similarly, let $y_{ab}$ denotes the received signal from ‘a’ to ‘b’. Then there are three received signals, $y_{sr}$, $y_{rd}$, and $y_{sd}$. The channel gain decays depending on the distance between the transmitters and receivers with a path loss fall off exponent, $\alpha$. Now, all the received signals are expressed in channel gain.
\[
y_w = \frac{1}{\alpha_1} x_w + n_1, \quad y_{rd} = \frac{1}{\alpha_2} x_{rd} + n_2, \quad y_{sd} = x_{sd} + n_3 \quad (1)
\]

Assume that noises, \( n_1, n_2, \) and \( n_3 \), are zero-mean independent Gaussian noise with unit power. Also, like normalized unity distance, the cooperation channel has a normalized bandwidth of 1Hz. Average total power constraint is also used, i.e.,
\[ E\left[ x^2_w + x^2_{rd} + x^2_{sd} \right] \leq P. \]

To see the effect of cooperation, analysis will be focused on obtaining the achievable sum-rate of given model, \( R_{sd} + R_{rd} \), where \( R_{sd} \) and \( R_{rd} \) denote the rate from source to destination and the rate from relay station to destination respectively. It shows us that how much transmitter cooperation improves the rate of systems. Furthermore, the result gives criterion between cooperation strategies.

### 3. ACHIEVABLE RATE FOR TRANSMITTER COOPERATION

In this section, we introduce two transmitter cooperation schemes, Pure Relay (PR) and Time Division Successive Broadcasting (TDSB), based on system model in section 2. Achievable rate of two schemes is showed as a function of location of relay station (R). To investigate the effect of cooperation with R, the rate of direct transmission without R, 1Tx 1Rx model, is used as a minimum boundary of two schemes.

#### 3.1. Pure Relay (PR)

A Pure Relay (PR) scheme is illustrated in Figure 3. In PR scheme, we assume that relay station forwards its received signal, \( R_{sr} \), as soon as detecting. Source uses power \( aP \), for \( a \in [0,1] \), to send a message to relay station with rate \( R_{sr} \), and relay station uses power \( (1-a)P \) to send a signal to destination with rate \( R_{rd} \) for total power constraint. Then the rate can be calculated as:

\[
R_{sr} = \log \left( 1 + \frac{aP}{r_1^a} \right) \quad (2)
\]

\[
R_{rd} = \log \left( 1 + \frac{(1-a)P}{r_2^a} \right) \quad (3)
\]

The relay station can not forward faster than the rate, which indicates received data from source, and so, total achievable rate of this scheme is:

\[
R_T = \max_{\theta \in [0,\pi]} (\min(R_{sr}, R_{rd})) \quad (4)
\]

Now, consider the achievable rate for PR scheme by equaling \( R_{sr} \) and \( R_{rd} \). For this equal rate condition, \( a \), power fraction of source and relay station is expressed as a function of position of relay station, \( r_1 \) and \( r_2 \):

\[
R_T = \max_{a=1-r_2} (\min(R_{sr}, R_{rd})), \quad a = \frac{r_1^a}{r_1^a + r_2^a} \quad (5)
\]

The achievable rate of PR cooperation scheme is shown in Figure 4 and Figure 5. Relay station is assumed to position at \((r_1, \theta)\) on polar coordinate. Used fall off exponent is 2, free space model.

Figure 4 presents the PR sum-rate capacity with fixed \( \theta \) and variable \( r_1 \). Figure 5 presents the rate with fixed \( r_1 \) and variable \( \theta \). Two graphs have similar trends; when relay station is on center of path, \( r_1 \) is equal to \( r_2 \) in detail, the rate is maximized. The reason can be explained easily from analysis of a graph at \( \theta = 0 \). If \( r_1 \) is smaller than 0.5, relay station would be located near the source. Then, the power to send a message from source to relay station would be smaller than the power to send a message from relay station to destination. On the other hand, if \( r_1 \) is bigger than 0.5, power to send from relay station to destination, would be relatively big. As a result, the achievable rate has the biggest value, when relay station is at center of source and destination since power consumption is the most effective. The reason why maximum rate is varied in Figure 4 is \( \theta \) variation. In other words, the point to make \( r_1 \) is equal to \( r_2 \) is changed with \( \theta \) variation.

Figure 5 shows the achievable rate with fixed \( r_1 \) and variable \( \theta \). Principle used in Figure 4 analysis is applied to this figure again. When \( r_1 \) is equal to 0.5, maximum achievable rate is obtained, because relay station is at almost center of source and destination. We see that our analysis is right from the graph at \( r_1 = 1 \). When \( r_1 \) is equal to
or greater than 1, relay station is useless, because relay station is farther than destination from S. As a result, the rate, when \( r_1 = 1 \), is lower than lower bound, 1Tx 1Rx model.

\[
R_{sr} = u \log \left( 1 + \frac{\mu aP}{u r_1^\alpha} \right) \quad (6)
\]

\[
R_{sd} = u \log \left( 1 + \frac{(1 - \mu)P}{u + \mu aP} \right) \quad (7)
\]

We found that addition of relay station improves the sum-rate capacity. However, we assumed that relay station sends its received signal as soon as detecting signal. This can not be realized, because relay station would have delay to process its received signal. This will decrease the rate. Therefore, we need to consider this problem by designing another scheme, which transmits signal not only through relay station but also without using relay station. Next TDSB model is one of them.

### 3.2. Time Division Successive Broadcasting (TDSB)

In this section, we consider another cooperation scheme Time Division Successive Broadcasting (TDSB) as illustrated in Figure.6. In TDSB, source broadcasts its own message to relay station and destination for \( u \) time slot, by using power \( aP/u \), for \( u, a \in [0, 1] \). The power fraction between \( R_{sr} \) and \( R_{sd} \) is assumed \( \mu \) and \( 1-\mu \), for \( \mu \in [0, 1] \), respectively. Now, the rate of first time slot can be obtained by

\[
R_{sr} = (1-u) \log \left( 1 + \frac{\mu aP}{(1-u)r_1^\alpha} \right) \quad (8)
\]

\[
< (1-u) \text{ time slot } >
\]

In TDSB model, destination receives data from two paths, from source and from relay station. Like PR model, relay station can not forward faster than the rate, which indicates received data from source, so,

\[
R_{rd} = \max \left( R_{rd} + \min( R_{sr}, R_{rd} ) \right) \quad (9)
\]

We set that \( R_{sr} \) is equal to \( R_{rd} \) and \( u, \) time division, is equal to 0.5. Power fraction \( \mu \) is equal to 0.5, too. This assumption makes \( R_{rd} \) to be maximized. Next, suppose that successive interference cancellation (SIC) is possible. The denominator of \( R_{sr} \) has an interference term, \( \mu aP \). If SIC is possible in relay station, this term could be cancelled. Then maximum sum-rate capacity is:

\[
R_T = \max_{\mu, u, a \in [0,1]} \left( R_{sd} + \min( R_{sr}, R_{rd} ) \right) \quad (10)
\]

We set that \( R_{sr} \) is equal to \( R_{rd} \) and \( u, \) time division, is equal to 0.5. Power fraction \( \mu \) is equal to 0.5, too. This assumption makes \( R_{rd} \) to be maximized. Next, suppose that successive interference cancellation (SIC) is possible. The denominator of \( R_{sr} \) has an interference term, \( \mu aP \). If SIC is possible in relay station, this term could be cancelled. Then maximum sum-rate capacity is:

\[
R_T = \max_{\mu, a \in [0,1]} \left( R_{sd} + \min( R_{sr}, R_{rd} ) \right), \quad a = \frac{r_1^\alpha}{r_1^\alpha + r_2^\alpha} \quad (11)
\]

Figure.7 and Figure.8 illustrate the achievable rate of TDSB scheme with fixed \( \theta \) and with fixed \( r_1 \) respectively. They show very similar trends with PR case. The rate of PR and TDSB is compared on Figure.9. Rate improvement of PR is better than that of TDSB. The reason is that PR scheme uses relay station more than TDSB scheme, which
use a power for direct transmission, too. However, TDSB model can give possibility of diversity gain, because it receives a message from two different channels. This can be another issue to research.

4. CONCLUSION

We have shown the achievable rate of two cooperation schemes, PR and TDSB in 2-dimensional space under total average power constraint. Addition of relay station results in improvement of rate. Moreover, TDSB gives an opportunity of diversity gain. However, we found that there may be two problems, delay from relay station and existence of obstacle in LOS path. These problems need to be resolved to realize these cooperation schemes in practical systems.

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6. REFERENCES


