Channel Estimation for Efficient Symbol Estimation Algorithm in High Mobility OFDM System

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ABSTRACT

In Orthogonal Frequency Division Multiplexing (OFDM) systems, the effect of intercarrier interference (ICI) caused by time-selectivity causes severe performance degradation and can not be eliminated. Among the symbol estimation schemes to mitigate effect of ICI, Iterative Sequential Neighbor Search (ISNS) algorithm can achieve enhanced performance with a moderate complexity. It is assumed to know perfect CSI. Considering practical case, we implement ISNS algorithm with newly estimated channel. The proposed channel estimator is advanced form of Least-squares (LS) channel estimator. By seeking full ICI terms and reducing noise effect in channel matrix, the performance is enhanced.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is attended as an attractive modulation scheme. In future broadband applications, its robustness and efficiency are important advantages for transmitting data at high data rates [1]. In time-frequency-selective OFDM systems, intersymbol interference (ISI) caused by frequency-selectivity of the channel can be prevented by a guard interval (GI) that is larger than maximum channel delay spread. However, in a time-selective fading channel, intercarrier interference (ICI) occurs, causing degrades the performance of the system. As the carrier frequency, block size, and vehicle velocity increase, the effect of ICI becomes more serious. Therefore, in time-selective fading channel, mitigation of the ICI caused by channel variations is necessary.

There are several approaches to reduce the ICI effect. Most of them require the complexity more than $O(N^2)$ operations. However, in [3], an efficient OFDM symbol estimation algorithm that has approximate $O(N^2)$ complexity is proposed. It is called Iteratively Sequential Neighbor Search (ISNS) algorithm. This algorithm roughly removes ICI terms and then looks for most similar symbol with temporary detected symbols by iteration. It can reduce complexity but also enhance performance with approximate $O(N^2)$ complexity.

On the other hand, the ICI reduction schemes that are introduced above are all assumed to know perfect channel state information (CSI). However, in real communication, receiver that actually estimates transmitted symbol can’t know any channel information. Therefore, in this paper, we implement practical symbol estimation with new channel estimation scheme in time-frequency-selective OFDM systems. This scheme is advanced form of Least-squares (LS) channel estimator [4] and more complex than LS estimator. However, we can show the outstanding performance enhancement compared with LS. We apply this estimated channel to the ISNS method to reduce computation complexity with enhanced performance.

In Section 2, we describe the system model. Section 3 discusses OFDM symbol estimation schemes; zero-forcing (ZF) method, ICI reduction ZF method, and ISNS that we use as symbol estimation scheme in simulation. In Section 4, channel estimation schemes, LS and new estimation method, are introduced. We simulate our channel estimation scheme with ISNS in Section 5.

Notation—A bold face letter denotes a vector or a matrix; $[\cdot]^\dagger$ denotes conjugate transpose; $[\cdot]$ denotes pseudo-inverse; $I_N$ denotes $N \times N$ identity matrix; $D_p(A)$ is the diagonal matrix including up to $p$th sub-diagonal and super-diagonal with the same term as matrix $A$; $M(A)$ is a matrix with the elements of $A$ on its diagonal;
in general, a lower case letter stands for a time-domain signal while an upper case letter denotes a frequency-domain signal.

2. OFDM SYSTEM DESCRIPTION

The discrete-time baseband equivalent OFDM system model is illustrated in Fig. 1. Input binary bits are coded into frequency-domain symbol, and then symbols are transferred by the serial-to-parallel converter. Then it is implemented by an inverse fast Fourier transform (IFFT).

The $i^{th}$ OFDM symbol block $X^i_F = \left[ X^i_0, X^i_1, ..., X^i_{N-1} \right]^T$ is converted to the time domain signal by the N-point IFFT.

$$X^i_T = F_N^* X^i_F,$$  

where $X^i_T = \left[ X^i_0, X^i_1, ..., X^i_{N-1} \right]^T$ and $F_N$ is an N-point IFFT matrix. The $i^{th}$ received block of time-domain OFDM samples

$$R^i = \left[ r^i_0, r^i_1, ..., r^i_{N-1} \right]^T = H X^i_T + Z^i,$$  

where $Z^i = \left[ z^i_0, z^i_1, ..., z^i_{N-1} \right]$ denotes the vector of time-domain white complex Gaussian noise (AWGN).

![Fig. 1. Baseband equivalent block diagram for an OFDM system](image)

The complex matrix, assumed to be perfectly known to the receiver, is given by

$$H = \begin{bmatrix} h^i_{0,0} & h^i_{0,N-1} & \cdots & h^i_{0,N-1} \\ h^i_{1,0} & h^i_{1,1} & \cdots & h^i_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ h^i_{N-1,0} & h^i_{N-1,1} & \cdots & h^i_{N-1,N-1} \end{bmatrix},$$  

where $h^i_{n,l}, 0 \leq n \leq N-1, 0 \leq l \leq L-1$, is the channel impulse response (CIR) during the $i^{th}$ OFDM symbol block interval. The received time-domain samples are transformed into frequency-domain symbols by N-point FFT operation. The demodulated OFDM symbol block is expressed as

$$Y = \left[ Y^i_0, Y^i_1, ..., Y^i_{N-1} \right] = F_N^* H F_N X^i_T + Z^i = G X^i_T + Z^i,$$  

where $F_N^*$ is the N-point FFT matrix, $G = F_N^* H F_N$ is an $N \times N$ matrix. The demodulated frequency-domain symbols can be expressed as

$$Y^i_z = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} X^i_{l} H^i_{k,l} e^{-j 2 \pi k l / N} + Z^i_z,$$  

and $Z^i_z$ is frequency-domain AWGN vector.

3. OFDM SYMBOL ESTIMATION SCHEMES

Frequency-selectivity of the channel causes ISI. ISI can be simply eliminated by inserting GI in front of the transmitted block that is longer than the maximum delay spread and copy of the last parts of the block for maintenance of orthogonality. However, ICI caused by time-selective fading channel leads to an error floor in BER. It becomes serious as the relative Doppler frequency $f_D$, which is the product of the Doppler frequency $f_D$ and OFDM symbol block duration $T_s$, increase. The most power of ICI is concentrated in the neighborhood of the desired subchannel and gradually decreases apart from it. By using this characteristic, reduced complexity symbol estimation schemes removing ICI terms are proposed.

3.1. Conventional methods

The zero-forcing (ZF) method is given as

$$\hat{X}_{ZF} = \left[ G^* \right] Y_x.$$

In time-invariant channel, the $G$ has only diagonal term, that is no ICI, therefore, the computation complexity of ZF is only $O(N)$. However, in time-variant channel, that we are interested in, (6) requires non-trivial inversion of the $N \times N$ matrix with $O(N^3)$ operations.

Since most power of ICI is concentrated in the neighborhood of the diagonal line of $G^*$, several terms that are merely zero to $Y^i$ can be neglected as [1]

$$g^i_{n,k} = 0, |n-k| > p,$$

where $2p$ denotes the number of dominant ICI terms against $n^{th}$ frequency-domain symbol. By using $D_p(G^*)$ instead of $G^*$, the complexity of ZF symbol estimation method is reduced. It is given as follows

$$\hat{X}_{ZF} = \left[ D_p (G^*) \right] Y_x.$$

3.2. Iterative Sequential Neighbor Search (ISNS)

ISNS symbol estimation algorithm is proposed in [3]. In this method, the following OFDM symbol error metrics are used.

$$\zeta^i_n = \left| Y^i_z - \sum_{k=0}^{N-1} g^i_{n,k} \bar{X}^i_k \right|,$$

**STEP 1.** Obtain an initial OFDM symbol candidate by

$$\tilde{X}^i_{(1)} = \left[ D_p (G^*) \right] Y_x.$$

**STEP 2.** Calculate $\zeta^i_n, 0 \leq n \leq N-1$, using $\tilde{X}^i_{(r)}$ with an iteration number $r$. 


STEP 3. Start to search the neighboring symbols in the constellation and update as follows

For \( n = 0 : N - 1 \)
- Search the neighbor symbols \( \{\hat{X}_{nt}^{(s)}\} \), \( 1 \leq s \leq S \), of \( \hat{X}_t^{(r)} \) in the constellation, where \( S \) is the number of candidate symbols \( \{\hat{X}_{nt}^{(r)}\} \).
- Calculate OFDM symbol error \( \hat{\xi}_n^{(r)} \) using the searched candidate symbols \( \{\hat{X}_{nt}^{(r)}\} \).
- If there exist \( \hat{\xi}_n^{(r)} \) such as \( \min_{1 \leq s \leq S} \hat{\xi}_n^{(s)} < \hat{\xi}_n^{(r)} \), update \( \hat{\xi}_n^{(r)} = \min_{1 \leq s \leq S} \hat{\xi}_n^{(s)} \) and \( \hat{X}_t^{(r)} = \hat{X}_{nt}^{(r)} \). Otherwise, remain the previous \( \hat{\xi}_n^{(r)} \) and \( \hat{X}_t^{(r)} \). End.

STEP 4. Repeat STEP 3 with \( r \leftarrow r + 1 \) to get more updated \( \hat{\xi}_n \) and \( \hat{X}_t^{(r)} \).

STEP 5. The end of the algorithm.

The initially estimated \( i \)th OFDM symbol block is obtained through the simple inversion. The estimator sequentially searches the neighboring symbol with minimum error metric in the constellation and updates \( \hat{X}_t^{(r)} \) and \( \hat{\xi}_n^{(r)} \).

STEP 3 is done for each subcarriers. The iteration number \( r \) is predetermined value. We should choose neighbor symbol areas that searches in each iteration. In Fig. 2., the area of neighbor search in the 16-QAM constellation is shown. The complexity of the algorithm increases as the neighbor search area increases.

![2nd Neighbor area](image)

4. OFDM CHANNEL ESTIMATION

In the real communication systems, the channel information is not known to the receiver. Therefore, to detect the received signal at the receiver, channel estimation needed. In this section, we will propose a channel estimation scheme.

4.1. Conventional method

The LS estimator is the simplest estimator. It is also called as zero-forcing estimator. This method doesn’t need any information of the channel. The estimated frequency-domain channel \( \mathbf{G}_e^{(r)} \) is expressed as

\[
\mathbf{G}_e^{(r)} = \mathbf{M} \left[ \mathbf{M} \left( \mathbf{X}_r \right) \right] \mathbf{Y}', \quad (10)
\]

where \( \mathbf{Y}' \) is demodulated symbol block that is expressed in (5) and \( \mathbf{G}_e^{(r)} \) is \( N \times N \) matrix that has only super-diagonal term. The computational complexity is only \( O(N) \).

Although LS estimator is very simple, it also has drawbacks. The BER performance of the estimator is seriously bad. The performance of LS will be shown in Section 5 by computer simulation.

4.2. Proposed channel estimation method

We propose a new channel estimation method to enhance the performance of the system. Proposed method use training blocks. In this paper, we use one data symbol block between two training blocks. If you want to increase data rate, you can use more data symbol blocks between training blocks. The channel of data symbol block is derived from interpolation between two side training blocks’ information.

This method starts with LS estimator. Estimated frequency-domain channel matrix \( \mathbf{G}_e^{(r)} \) in (10) has only super-diagonal term because its components is made by division between each components of demodulated OFDM symbol block and training symbol block that we know.

Considering, \( \mathbf{G}' = \mathbf{F}_r^H \mathbf{H} \mathbf{F}_r \), we can get estimated time-domain channel matrix \( \mathbf{H}_e^{(r)} \) from estimated frequency-domain channel matrix \( \mathbf{G}_e^{(r)} \). Because, \( \mathbf{F}_r^H \mathbf{F}_r = \mathbf{F}_r^H \mathbf{F}_r = \mathbf{I}_r \)

\[
\mathbf{H}_e^{(r)} = \mathbf{F}_r^H \mathbf{G}_e^{(r)} \mathbf{F}_r, \quad (11)
\]

where \( \mathbf{F}_r \) and \( \mathbf{F}_r^H \) are \( N \)-point FFT and IFFT matrix.

Looking into \( \mathbf{H}_e^{(r)} \), \( \tilde{h}_{0,0} \sim \tilde{h}_{N-1,0} = \alpha \), \( \tilde{h}_{n,0} \sim \tilde{h}_{N-1,n} = \beta \), \( \tilde{h}_{0,N-1} \sim \tilde{h}_{N-1,N-1} = \gamma \), where \( \alpha, \beta, \gamma \) are constant value.

That is, it is because \( \mathbf{G}_e^{(r)} \) has only super-diagonal term which means there is no time-varying effect. It is a limitation of LS estimator. Although the time-selective fading channel seriously effects to the transmitted signal, LS can’t detect those effects. Therefore, we attempt to get full matrix of frequency-domain channel matrix \( \mathbf{G}_e^{(r)} \) that contains ICI terms which represents there exist time-selectivity in the channel.

To detect the information of delay profile, we remain the diagonal line in \( \mathbf{H}_e^{(r)} \) that has value bigger than threshold. We set threshold as \( 1/10 \) times of transmit power. The points that diagonal lines remain represent the points that delay spread exists. In other words, we can remain the diagonal lines that are matched to the delay profile and do nulling to other lines. By doing this, we can eliminate the
AWGN effect in \( \mathbf{H}^i \). Now then, we get \( \mathbf{H}^{i+2} \), and then remain the diagonals that is identical to \( \mathbf{H}^i \).

For example, if we use 2-ray Rayleigh fading channel as the relative delays at first and third symbol duration, and the FFT/IFFT size \( N=4 \), \( \mathbf{H}^i \) is described as

\[
\mathbf{H}^i = \begin{bmatrix}
\alpha' & 0 & \beta' & 0 \\
0 & \alpha' & 0 & \beta' \\
\beta' & 0 & \alpha' & 0 \\
0 & \beta' & 0 & \alpha'
\end{bmatrix},
\]

where \( \alpha \) and \( \beta \) are the complex channel gain at first and third symbol duration, respectively. Now then we get \( \mathbf{H}^{i+2} \)

\[
\mathbf{H}^{i+2} = \begin{bmatrix}
\alpha'' & 0 & \beta'' & 0 \\
0 & \alpha'' & 0 & \beta'' \\
\beta'' & 0 & \alpha'' & 0 \\
0 & \beta'' & 0 & \alpha''
\end{bmatrix},
\]

and \( \mathbf{H}^{i+1} \) is estimated by interpolation between values of \( \mathbf{H}^i \) and \( \mathbf{H}^{i+2} \).

\[
\mathbf{H}^{i+1} = \begin{bmatrix}
\alpha_i & 0 & \beta_i & 0 \\
0 & \alpha_i & 0 & \beta_i \\
\beta_i & 0 & \alpha_i & 0 \\
0 & \beta_i & 0 & \alpha_i
\end{bmatrix},
\]

where \( \alpha_1 \sim \alpha_4 \) are linear interpolated value between \( \alpha' \) and \( \alpha'' \) and \( \beta_1 \sim \beta_4 \) is obtained in the same way. We can extend this application into other OFDM systems that have different conditions. We can use more data symbol blocks between training blocks, and should get new delay profile information regularly. Finally, we get \( \mathbf{G}^{i+1} \) that we want to actually know from \( \mathbf{H}^{i+1} \).

By removing the noise effect in \( \mathbf{H} \) and doing interpolation, we can measure full matrix of \( \mathbf{G} \) that contains full ICI terms. New estimation method is somewhat complex and has lower data rates but outperforms than LS.

5. SIMULATION RESULTS

To evaluate the performance of the ISNS with proposed channel estimation scheme, computer simulations are carried out.

5.1. Simulation environment

We use 16-QAM as a modulation and the FFT/IFFT size \( N \) is 64. The symbol block duration \( T_s=128\mu s \), which means each symbol duration is 2\( \mu s \), maximum channel delay is 4\( \mu s \), and the GI duration is 6\( \mu s \). We use Jake’s model with a random phase as a channel model and it is for the 2-ray Rayleigh fading channel with 0 and 4\( \mu s \) relative delay profile. Each delay components has 0dB power. We choose the first neighbor area as the search area.

5.2. Simulation results

We will show the computer simulation results about BER performance of ISNS symbol estimation algorithm, proposed channel estimation scheme, and ISNS symbol estimation with proposed channel estimation method.

Fig. 3. BER performance of ZF method and ISNS algorithm for various relative Doppler frequency

We compute conventional method ZF with full matrix \( \mathbf{G} \) and diagonal matrix \( \mathbf{D}_r(\mathbf{G}) \) to compare the performance with ISNS algorithm computed for various diagonal line and iteration number. As can be seen in Fig. 3., outstanding gain of ISNS algorithm with \( \mathbf{D}_r(\mathbf{G}) \) is achieved compared to the conventional ZF with \( \mathbf{D}_c(\mathbf{G}) \).

The performance enhancement of ISNS algorithm is obtained by the number of iteration and calculation of the OFDM symbol errors which increase complexity. However, the resultant complexity is reasonable, since it is still approximately \( O(N^2) \). The complexity of ISNS algorithm increases as iteration number \( r \) and considered dominant neighboring subcarrier number \( p \) increase, respectively.

Fig. 4. shows system performance of using LS estimator and proposed estimator. Comparing with LS
estimator, our proposed estimation scheme has better system performance.

![Fig. 4. BER performance of ZF method with LS estimator and proposed estimation scheme with various relative Doppler frequency](image)

This improvement is obtained by seeking all ICI terms of frequency-domain channel matrix $G$ and eliminating noise effect in time-domain channel matrix $H$.

Lastly, we simulate ISNS algorithm with our estimated channel. The results are represented in Fig. 5. We use 9 symbol blocks in simulation which means 5 blocks for training and 4 blocks for transmitting data. We realize the practical system by simulating symbol estimation scheme with estimated channel.

![Fig. 5. BER performance of ISNS symbol estimation algorithm with estimated channel for various relative Doppler frequency](image)

6. CONCLUSION

OFDM systems over a time-frequency-selective fading channel result in ICI that causes severe performance degradation. There are many methods to mitigate ICI effect, among them ISNS algorithm provides good performance with low computational complexity [3]. Considering receiver doesn’t have any information about the channel, we use newly estimated channel in ISNS algorithm. The proposed estimated method is advanced form of LS and outperforms it. The performance enhancement is earned by seeking ICI terms of frequency-domain channel matrix and mitigating AWGN effect in time-domain channel matrix. However, new scheme is somewhat complex and has low data rate.

7. REFERENCES