Digital Timing Error Correction in Discrete Multitone Systems

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SUMMARY When a fixed free-running crystal is used for sampling time generation at a DMT receiver, Inter-Symbol Interference (ISI) and Inter-Carrier Interference (ICI) are introduced by sampling time error. The ICI becomes more serious as the DMT symbol length increases. In this letter, the effects of sampling clock offset are investigated in the time domain using a new notion of Inter-sample Interference (IsI) instead of ISI and ICI. Based on the IsI analysis, we propose a new frequency domain timing error correction scheme.

key words: non-synchronized sampling, discrete multitone

1. Introduction

Discrete multi-tone (DMT) is a modulation technique used for data transmission over subscriber lines as in ADSL and VDSL systems [1], [2]. There are two possible sampling schemes at the DMT receiver. One is a synchronous sampling scheme and the other is non-synchronized. In synchronous sampling systems, a voltage controlled oscillator (VCO) is used to determine the sampling time using an estimated clock offset. However, in non-synchronized sampling systems using a fixed free-running crystal, the timing error correction is done in the digital domain without feedback to the analog domain.

This letter addresses non-synchronized sampling DMT systems. The effect of sampling time error is serious for non-synchronized sampling DMT systems especially when a long DMT symbol is used. The serious effect of sampling time error on non-synchronized sampling multi-carrier systems was addressed in [3].

A digital timing correction scheme combining temporal and spectral signal properties was proposed [4], [5]. In the previous approach, time domain interpolation is used in conjunction with the frequency domain rotor to resolve the sampling frequency offset (SFO) problem. However, the time domain interpolation approach requires over-sampling at the analog front end, which is not suitable for high-speed systems such as VDSL systems. Furthermore, the time domain/frequency domain hybrid approach assumes moderate DMT symbol length so that only a small fraction of the timing error is to be corrected by time domain interpolation and the big symbol alignment is done in the frequency domain.

Unfortunately, however, longer DMT symbols are used for VDSL systems [2]. In this letter, a new computationally efficient frequency domain timing error correction scheme without time domain interpolation is proposed. Moreover, the proposed approach works efficiently regardless of the DMT symbol length. Computer simulations are performed to demonstrate the efficacy of the proposed approach.

2. Timing Error in Non-synchronized Sampling DMT Systems

The DMT time domain signal at the receiver can be expressed as

\[ r(t) = \frac{1}{2N} \sum_{m=-\infty}^{\infty} \sum_{k=0}^{2N-1} a_m^k g_m^k(t) + n(t) \]

with

\[ g_m^k(t) = \sum_{n=0}^{2N-1} g(t - (m(2N + \nu) + n)T) e^{j2\pi kn} \]

where \( g(t) \) stands for the composite channel impulse response including the transceiver filters, equalizer and channel; \( \nu \) is the cyclic prefix length; \( a_m^k = (a_m^{2N-1})^*, 1 \leq k \leq N - 1 \), is the quadrature amplitude modulation (QAM) symbol at the \( k \)-th carrier in the \( m \)-th DMT symbol where \( (\cdot)^* \) denotes complex conjugate; \( N \) is the number of carriers in the DMT system; \( T \) is the sample period at the transmitter; and \( n(t) \) is the additive noise [5]. This continuous time signal is inputted to the receiver sampling device. When the free running oscillator incurs SFO, system performance is degraded.

The effect of SFO is evaluated in terms of Inter-sample Interference (IsI) instead of ISI and ICI. If we assume no interference between samples due to the channel, the time domain samples of the \( m \)-th DMT symbol with SFO \( \delta_m \) can be expressed as

\[ s_m^m = \frac{1}{2N} \left[ \sum_{k=0}^{N-1} a_m^k e^{j2\pi \left[ \frac{m+\delta-(m(2N+\nu)+\nu+\delta+n)T}{N} \right]} \right] + \sum_{k=0}^{2N-1} a_m^k e^{-j2\pi (2N-\nu)(m+\delta-(m(2N+\nu)+\nu+\delta+n)T/2N)} \]

where \( \Delta f \) is the SFO and \( \delta \) is an integer such that \( |\epsilon_m^0| \leq 0.5 \) where \( \epsilon_m^0 \) is given as \( \delta - [m(2N + \nu) + \nu] \Delta f / T \). If this
received symbol is aligned by the frequency domain rotor whose angle is $-2\pi\frac{m}{N}(1 - \frac{m}{N}) - [m(2N + v) + N + 0.5] \frac{2\pi}{N}$, then the estimated time domain symbol is given as

$$s_m^e = \frac{1}{2N} \left[ \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}[n-(n-(N+0.5))\frac{N}{N}]} + \sum_{k=N}^{2N-1} a_k e^{-j\frac{2\pi}{N}[n-(n-(N+0.5))\frac{N}{N}]} \right]$$  \hspace{1cm} (4)$$

Let $a_k = \sum_{l=0}^{2N-1} s_l e^{-j2\pi kl/N}$, then the estimated symbol (4) can be rewritten as

$$s_m^e = s_m^n I_{n,n} + \sum_{l=0,\text{even}}^{2N-1} s_l^n I_{l,n},$$  \hspace{1cm} (5)$$

where

$$I_{l,n} = \frac{1}{N} \cos \left[ \frac{\pi(N-1)}{2N} (n-l - (n-(N+0.5)) \frac{N}{f_s} \right]$$

$$\sin \left[ \frac{\pi}{2N} (n-l - (n-(N+0.5)) \frac{N}{f_s} \right] \sin \left[ \frac{\pi}{2N} (n-l - (n-(N+0.5)) \frac{N}{f_s} \right], \hspace{1cm} (6)$$

The variance of the ISI is proportional to the square of frequency offset and it is also approximately proportional to the square of $|N+0.5-n|$, the distance from the center point.

### 3. Timing Error Correction Schemes

#### 3.1 Previous Work

When SFO occurs, time domain interpolation as well as frequency domain rotation is necessary. An approach was proposed [5] where it is not necessary to interpolate for all the sampling time error. The timing error of the $n$-th sample in the $m$-th symbol can be written as

$$e_m^n = e_m^0 + n\Delta T = (e_m^0 + N\Delta T) + \Delta T(n-N),$$  \hspace{1cm} (8)$$

where $\Delta T$ is the difference between the sampling intervals of the transmitter and receiver. The time domain interpolator corrects the second term only and the first term, the average timing error of the $m$-th DMT symbol, is corrected by the frequency domain rotor. However, over-sampling at the analog front end is required when time domain interpolation techniques are used. The over-sampling is not desirable especially for high speed systems such as VDSL systems.

Furthermore, the second term of (8) to be corrected by the time domain interpolation grows in proportion to the symbol length. We note that as the second term grows, the more complex interpolation is required [6]. So the previous approach works properly only when moderate symbol length is assumed.

#### 3.2 A Proposed Approach

According to the analysis of timing error effects in section II, it can be seen that a portion (samples close to the center) of the time domain symbol can be recovered with sufficiently small ISI by frequency domain rotor when moderate SFO is assumed. In the proposed approach, only this portions are recovered iteratively.

The new approach rearranges the current block into $P$ sub-symbols in the time-domain. Let $s_m = [s_m^0, s_m^1, \ldots, s_m^{2N-1}]$ be the current block after the sampling with timing error. This block is rearranged into $P$ short sub-symbols. For example, when $P = 2$ the current block is rearranged into 2 sub-symbols as

$$s_{m1} = [s_m^0, s_m^1, s_m^2, \ldots, s_m^{2N-1}],$$

$$s_{m2} = [s_m^0, s_m^1, \ldots, s_m^{N-1}, s_m^N, \ldots, s_m^{N+1}, \ldots, s_m^{2N-2}, s_m^{2N-1}, s_m^0, \ldots, s_m^{N-1}, s_m^N, \ldots, s_m^{2N-1}],$$

$$\ldots$$

We note that the samples in the previous symbol ((m+1)-th) and the next symbol ((m+1)-th) are involved in the rearrangement. We also note that although the name is sub-symbol in the time domain, the length of each sub-symbol is $2N$, which is the FFT size. We name it sub-symbol because we are interested in only a portion of the sub-symbols. After the rearrangement, frequency domain timing error correction is done for each sub-symbol. This procedure can be described as follows

For each $p \in \{0, 1, \ldots, P-1\}$

$$Z_{mp,k} = DFT_{2N}(s_{mp})$$

Rotation of $Z_{mp,k}$ by $\bar{\epsilon}_{mp}$

$$Z_{mp,n} = I DFT_{2N}(Z_{mp,k})$$

where $\bar{\epsilon}_{mp}$ is the average timing error for the $p$-th partition of the $m$-th DMT symbol. The average timing error $\bar{\epsilon}_{mp}$ is expressed as

$$\bar{\epsilon}_{mp} = \frac{2N}{P} \sum_{m=0}^{2N-1} \sum_{p=0}^{P-1} \epsilon_m^{p}.$$  \hspace{1cm} (9)$$

After the correction, the estimated time domain DMT symbol $\hat{s}_m = [\hat{s}_m^0, \hat{s}_m^1, \ldots, \hat{s}_m^{2N-1}]$ is given as

$$\hat{s}_m = [\bar{z}_m^0, \bar{z}_m^1, \ldots, \bar{z}_m^2, \bar{z}_m^{2N-2}, \bar{z}_m^{2N-1}, \bar{z}_m^0, \ldots, \bar{z}_m^{N-1}, \bar{z}_m^N, \ldots, \bar{z}_m^{2N-1}, \bar{z}_m^0, \ldots]$$

\ldots
\[ z_{mP} \frac{-2\pi}{\nu P} \cdot 1 \cdot \cdots \cdot z_{mP} \frac{-2\pi}{\nu P} \cdot 1 \]  

Finally, \( \hat{S}_m \) is fed to the demodulating FFT, producing estimated QAM symbols. The proposed approach is computationally efficient because it does not involve either oversampling at the analog front end or time domain interpolation. Furthermore, because the long time domain DMT symbol is iteratively recovered, this approach works efficiently regardless of the length of the DMT symbol.

4. Simulation Experiment Results

In this section, the performance of the proposed scheme is demonstrated through simulation. The FFT size is 512 or 1024. A cyclic prefix of 32 samples is used. The values, \( P = 4 \) when \( N = 256 \) and \( P = 8 \) when \( N = 512 \), are used for the proposed approach. A 16-QAM constellation is used for symbol mapping in each sub-channel. An SNR of 30 dB in the time domain is assumed. Correct timing error detection is also assumed. A raised cosine filter with roll off factor of 0.03 is used as the transmit and receive filter. The performance criterion is the normalized mean square error (NMSE) defined as

\[ \text{NMSE} = \frac{E[|s_m^n - \hat{s}_m^n|^2]}{E[|s_m^n|^2]} \]

where \( s_m^n \) denotes the true value and \( \hat{s}_m^n \) denotes the estimated value.

Figure 1 compares the NMSE’s when \( N = 256 \) and a SFO is 100 ppm. As seen in the figure, the DMT symbol is not properly recovered when the frequency domain rotor is used. Only a fraction around the center point in the time domain is recovered in accordance with (8). The proposed scheme with \( P = 4 \) reduces the ISI terms in (5) to a proper level.

Figure 2 compares the recovered constellations corresponding to NMSE’s of Fig. 1. Figure 2(a) is the constellation recovered by the frequency domain rotor and Fig. 2(b) is the constellation recovered by the proposed scheme with \( P = 4 \). We can see that the proposed scheme recovers the constellation almost perfectly.

Figure 3 shows NMSE’s when \( N = 512 \) and SFO = 100 ppm. The effect of the time domain symbol length can be observed. The NMSE after the frequency domain rotation when \( N = 512 \) is much higher than when \( N = 256 \) with SFO fixed at 100 ppm. But the proposed scheme with \( P = 8 \)

![Fig. 1](image1.png)  
*Fig. 1* NMSE comparison when \( N = 256 \) and SFO=100 ppm.

![Fig. 2](image2.png)  
*Fig. 2* Recovered constellation when \( N = 256 \) and SFO=100 ppm (a) frequency domain rotor. (b) proposed approach with \( P = 4 \).

![Fig. 3](image3.png)  
*Fig. 3* NMSE comparison when \( N = 512 \) and SFO = 100 ppm.
reduces the ISI in terms of NMSE to the level of less than −30 dB for the entire time domain symbol again.

Figure 4 shows the recovered constellations corresponding to NMSE’s of Fig. 3. Figure 4(a) corresponds to the case when the frequency domain rotor is used and Fig. 4(b) is when the proposed approach with $P = 8$ is applied to the samples with timing error. The proposed approach with $P = 8$ recovers the constellation almost perfectly again. These results suggest that the proposed approach successfully compensates for the timing error regardless of the DMT symbol length.

5. Conclusions

In this letter, a new computationally efficient timing error correction scheme was proposed for non-synchronized sampling DMT systems. Although the new approach introduces a one block delay it is computationally efficient in that neither over-sampling at the analog front end nor time domain interpolation is involved. Furthermore, the proposed approach successfully mitigates the effect caused by timing error regardless of the DMT symbol length.

Acknowledgments

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References