Efficient Indoor Ranging Algorithm using Chirp Signalling

Na Young Kim, Sujin Kim, †Jaehwan Kim, Jeongsu Lee, and Joonhyuk Kang

School of Engineering, Information and Communications University,
119 Munjiro Yuseong-gu, Daejeon, 305-732, Korea
E-mail: {nykim, kksj0613, leon0035, and jhkang}@icu.ac.kr
†Electronics and Telecommunications Research Institute,
138 Gajeongno, Yuseong-gu, Daejeon, 305-700, Korea
E-mail: kimj@etri.re.kr

Abstract—In this paper, we propose an efficient ranging technique using chirp signal, which employs a matrix pencil method to estimate time of arrival (TOA). The proposed TOA estimation method consists of a two-step signal processing: minimum mean square error (MMSE)-based channel impulse response estimation and the channel delay tracking by matrix pencil algorithm. At the first step, the transmitted known chirp signals are cancelled out using the MMSE criterion. Then, the delay components are extracted by matrix pencil method using the estimated frequency channel impulse response. Simulation results show that the proposed scheme achieves small ranging error with low complexity.

I. INTRODUCTION

A location finding technique has been paid considerable attention as the location-based applications such as enhanced 911 (E911) services [1] and intrusion protection system become wide-spread. Usually the services and systems require accurate location estimation especially in indoor environment. Prior to obtaining the accurate location, ranging information might be first needed. For instance, position location can be found by a trilateration method using three range information. Even 1 reference based location technique needs ranging information [2], [3]. Hence, we focus on the ranging technique based on a time of arrival (TOA) estimation.

In a multipath environment, only the shortest delay component is necessary for the TOA estimation. Ranging estimation algorithm from the TOA information has been researched with various techniques, such as multiple signal specification (MUSIC) [4], minimum-norm [5], [6], and total least square-estimation of signal parameter via rotational invariance techniques (TLS-ESPRIT) [7]. These super-resolution techniques can increase time-domain resolution. Then again, they also increase computational complexity of a system since they utilize the covariance matrix of the received signal samples.

In this paper, our main contribution is to provide an efficient ranging algorithm using the chirp signal. The proposed approach uses a minimum mean square error (MMSE) based channel impulse response estimation and the matrix pencil method for low complexity estimates of TOA information.

Simulation results show that for the suggested technique, the small ranging error can be achieved with less computational complexity, especially compared to conventional MUSIC algorithm.

The paper is organized as follows. Signal and system model is outlined in section II. Section III is devoted to the proposed TOA estimation technique including the sketch of matrix pencil. The results concerning the performance and complexity of the proposed scheme are presented in section IV. Finally, we reach the conclusion in section V.

Notation: Throughout this paper, bold symbols denote matrices or vectors. (·)T, (·)H, (·)+ denote transpose, Hermitian transpose, pseudo inversion operation respectively.

II. SYSTEM MODEL

We consider a transceiver structure for ranging estimation based on the TOA, as shown in Fig. 1. It is generally believed that the chirp spread spectrum (CSS) suits with high accuracy ranging application especially in indoor environment where the multipath components are dense. Thus, we consider the system which employs the chirp signal based on the IEEE standard 802.15.4a [13]. The sub chirp sequence with its associated
The signal generating, the transmitter produces a preamble by time gaps is generated from the following equation:

\[ s^m(t) = \sum_{n=0}^{\infty} s^m(t, s) \]

\[ = \sum_{n=0}^{4} \sum_{k=1}^{4} e^{j\omega_{k,m} + j\omega_m(t-T_{n,k,m})} (t-T_{n,k,m}) P_{RC}(t-T_{n,k,m}) \]

where \( m = 1, 2, 3, 4 \) defines which of the four different possible chirp symbols (sub-chirp sequences) is used, \( n = 0, 1, 2, \ldots \) is the sequence number of chirp symbols, \( P_{RC} \) is the raised cosine function and the other variables are

\[ \tilde{c}_{n,k} = e^{j\pi/4}, e^{-j\pi/4}, e^{j3\pi/4}, e^{-j3\pi/4} \]

\[ T_{n,k,m} = (k-0.5)T_{sub} + nT_{chirp} - (1 - (-1)^n) \tau_m \]

\[ T_{sub} = 1.1875 \times 10^{-6} \text{sec} \]

\[ T_{chirp} = 6.0 \times 10^{-6} \text{sec} \]

\[ \mu = \omega_{BW}/T_{sub} \]

\[ \omega_{BW} = 2\pi \times 6.95 \times 10^6 \times (1 + \alpha), \quad \alpha = 0.25 \]

\[ \tau_1 = 4.6875 \times 10^{-7} \text{sec}, \quad \tau_2 = 3.125 \times 10^{-7} \text{sec}, \]

\[ \tau_3 = 1.5615 \times 10^{-7} \text{sec}, \quad \tau_4 = 0 \text{sec} \]

\[ \omega_{k,m} = 2\pi \times f_{k,m} \]

The generated signal waveform is illustrated in Fig. 2. After the signal generating, the transmitter produces a preamble by repeating the signal eight times. The generated preamble is transmitted through the transmit antenna. At the receiver, the received signal after the multipath channel is expressed as

\[ r(t) = \sum_{k=0}^{L_p-1} \alpha_k s(t - \tau_k) + n(t), \]  

where \( L_p \) is the total number of multipath signals, \( \alpha_k \) and \( \tau_k \) are the amplitude and the propagation delay of the \( k \)-th path respectively, \( s(\cdot) \) is the transmitted signal shape and \( n(t) \) is the additive white Gaussian noise (AWGN). Eq. (2) can be represented in the frequency domain [11] as

\[ R(\omega) = \sum_{k=0}^{L_p-1} S(\omega) H(\omega) + N(\omega) \]

\[ = \sum_{k=0}^{L_p-1} \alpha_k S(\omega)e^{-j\omega\tau_k} + N(\omega). \]  

(3)

The discrete measurement data are obtained by sampling the channel frequency response at \( L \) equally spaced frequencies. The sampled discrete frequency domain received signal of the Eq. (3) is given by

\[ r(l) = S(l)H(f_l) + n(l) \]

\[ = \sum_{k=0}^{L_p-1} \alpha_k S(l)e^{-j2\pi(f_l + \Delta f)\tau_k} + n(l) \]  

(4)

where \( l = 0, 1, \ldots, L - 1 \), \( f_0 \) is center frequency, and \( \Delta f \) is the sampling interval in frequency domain. Eq. (4) can be expressed matrix formula as

\[ \mathbf{r} = \mathbf{S}\mathbf{h} + \mathbf{n} \]

\[ = \mathbf{SVa} + \mathbf{n}, \]  

(5)

where

\[ \mathbf{r} = [ r(0) \quad r(1) \quad \cdots \quad r(M - 1) ]^T, \]

\[ \mathbf{a} = [ \alpha_0' \quad \alpha_1' \quad \cdots \quad \alpha_{L_p-1}' ]^T, \]

\[ \mathbf{S} = [ \mathbf{s}(0) \quad \cdots \quad \mathbf{s}(L - 1) ], \]

\[ \mathbf{s}(k) = [ s(k) \quad \cdots \quad s(k + M - 1) ]^T, \]

\[ \mathbf{V} = [ \mathbf{v}(\tau_0) \quad \mathbf{v}(\tau_1) \quad \cdots \quad \mathbf{v}(\tau_{L_p-1}) ], \]

\[ \mathbf{v}(\tau_k) = [ 1 \quad e^{-j2\pi f_{\tau_k}} \quad \cdots \quad e^{-j2\pi(M-1)\Delta f_{\tau_k}} ]^T, \]

and \( N \) is the total number of symbols, \( M = N - L + 1 \). The noisy received signal is first averaged in order to obtain the effect of time diversity. After that, the channel impulse response (CIR) is estimated by employing the MMSE criteria. Then, it is possible to compute the distance between two devices using matrix pencil.

III. PROPOSED TOA ESTIMATION

In this section, we introduce the proposed TOA estimation method with two-step signal processing: MMSE-based cancelation and the shortest channel delay tracking by matrix pencil algorithm.

Before the signal processing, receiver firstly average the preamble signal which is known to the receiver. The averaging preamble make the noise be diminished by getting a sort of time diversity since the noise has Gaussian distribution.
A. MMSE Based Channel Impulse Response Estimation

Typically, the transmitted signals can be recovered by applying the inversion or pseudo inversion of the channel matrix in signal detection area. On the other hand, the pseudo inversion of the known signal matrix is needed here since the considered problem is to find the channel impulse response information. The channel impulse response can be easily obtained by multiplying both sides of (5) with inversion of the signal shape matrix $S^\cdot$. That is,

$$S^+ r = S^+ S H + S^+ n. \quad (6)$$
or

$$\tilde{r} = H + \tilde{n}. \quad (7)$$

Based on the MMSE criterion, $S^+$ is expressed as

$$S^+ = S^H \{ S \cdot S^H + (N_0/2) \cdot I \}^{-1}, \quad (8)$$

where $N_0$ is noise variance and $I$ represents an identity matrix. The estimated $H$ is applied to the TOA estimation using matrix pencil.

B. TOA Estimation using Matrix Pencil

The frequency response of the estimated noisy CIR from the Eq. (6) can be written as

$$H(j2\pi f \Delta f) = \sum_{k=0}^{L_p-1} \alpha_k z_k^l + n_k, \quad (9)$$

where $z_k = e^{-j2\pi f \Delta f \tau_k}$ with $\Delta f = 1/L \Delta f[8]$. Here, we should take notice that once $z_k$ is estimated, the delay component, $\tau_k$, and also TOA can be estimated.

Now the well-known matrix pencil algorithm to estimate the desired information $\tau_k$ is briefly introduced. We begin by defining estimated CIR matrix from Eq. (7) and (9) as a matrix pencil input $X$ which has $(L - P) \times (P + 1)$ dimension representing the $l$-th frequency sample at $l \Delta f$.

$$X = \begin{bmatrix}
    H(0) & \cdots & H(P) \\
    \vdots & \ddots & \vdots \\
    H(L - P - 1) & \cdots & H(L - 1)
\end{bmatrix}, \quad (10)$$

where $P$ represents the pencil parameter that should satisfy the following Eq. (11) [9]

$$L_p \leq P \leq L - L_p, \quad L : \text{even},$$
$$L_p \leq P \leq L - L_p + 1, \quad L : \text{odd}. \quad (11)$$

Next, let us define two $(L - P) \times P$ matrices $X_0$ and $X_1$, each of which consists of the first $P$ and last $P$ columns of $X$. These matrices can be expressed as

$$X_0 = Z_L A Z_R,$$
$$X_1 = Z_L A \Phi Z_R, \quad (12)$$

where

$$\begin{align*}
    r &= S H + n, \quad R_{\sum} = 0 \\
    &\text{for } i = \text{preamble} \\
    R_{\text{sum}} &= R_{\text{sum}} + R(\cdot, :) \\
    &\text{end} \\
    R_{\text{mean}} &= R_{\text{sum}} / \text{preamble} \\
    S_{\text{MMSE}} &= S^H \{ S \cdot S^H + (N_0/2) \cdot I \}^{-1} \\
    \mathbf{H} &= S_{\text{MMSE}} \cdot R_{\text{mean}} \\
    X_0 &= \mathbf{H}(\cdot, : P) \\
    X_1 &= \mathbf{H}(\cdot, : P + 1) \\
    z_k &= \text{GE}(X_0, X_1) \\
    \tau_k &= -\text{imag}[\text{ln}(z_k)]/2\pi \Delta f, \quad k = 1, \ldots, L_p
\end{align*}$$

Table I

<table>
<thead>
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<td>PSEUDO CODE OF THE PROPOSED SCHEME</td>
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</table>

where

$$Z_L = \begin{bmatrix}
    1 & 1 & \cdots & 1 \\
    z_1 & z_2 & \cdots & z_{L_p} \\
    \vdots & \vdots & \ddots & \vdots \\
    z_{(L - P - 1)} & z_{(L - P - 2)} & \cdots & z_{(L - P)} \end{bmatrix}_{(L - P) \times L_p}$$

$$Z_R = \begin{bmatrix}
    1 & z_1 & \cdots & z_{(P - 1)} \\
    z_2 & z_2 & \cdots & z_2 \\
    \vdots & \vdots & \ddots & \vdots \\
    z_{L_p} & z_{L_p} & \cdots & z_{L_p} \end{bmatrix}_{L_p \times P}$$

$$\Phi = \begin{bmatrix}
    z_1 & z_2 & \cdots & z_{L_p} \end{bmatrix}_{L_p \times L_p}$$

$\Phi$ is the diagonal matrix which we want to estimate. Without noise, if we choose the pencil parameter $P$ that satisfies the constraints in Eq. (11), the desired parameter can be obtained by solving the following matrix pencil

$$X_1 - \lambda X_0 = Z_L A [\Phi - \lambda I] Z_R, \quad (13)$$

where the matrices $X_0$ and $X_1$ have rank $L_p$. For an arbitrary $\lambda$, the matrix difference (Eq. (13)) also has rank $L_p$. However, if we choose $\lambda$ out of $z_l$, i.e., $\lambda = z_k$, for some $l \in [1, L_p]$, the rank of the matrix difference reduces by one to $L_p - 1$. Thus, we can find the $z_k$ as the generalized eigenvalues of the matrix pair $[X_1, X_0]$. The delay component $\tau_k$ can be evaluated by using

$$\tau_k = -\text{imag}[\text{ln}(z_k)]/2\pi \Delta f, \quad k = 0, 1, \ldots, L_p. \quad (14)$$

where imag is the imaginary part operator.

Finally, we can assess the range with multiplying $\tau_k$ by the propagation speed of an electric wave. Table I gives an outline of the proposed matrix pencil TOA estimation using chirp signalling.
### TABLE II
SHORT DESCRIPTION OF MUSIC AND THE PROPOSED MATRIX PENCIL ALGORITHMS

<table>
<thead>
<tr>
<th>MUSIC</th>
<th>Matrix Pencil</th>
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<tbody>
<tr>
<td>$R_{xx} = \sum_{n=1}^{L} x x^H$</td>
<td>$[\lambda, q] = \text{GE}(X_0, X_1)$</td>
</tr>
<tr>
<td>$[R_{qq}] = \text{EVD}(R_{xx})$</td>
<td>$P_{\text{MUSIC}} = \frac{1}{L-1} \sum_{k=1}^{L_p}</td>
</tr>
</tbody>
</table>

### IV. SIMULATION RESULTS

In this section, we desire to illustrate the validity of the proposed TOA estimation technique. Both the complexity and performance of the proposed algorithm are analyzed. Moreover, the comparison between the conventional super-resolution technique, especially MUSIC and the proposed ranging estimation algorithm is represented.

The chirp signal described in Eq. (1) is generated with the raised cosine window pulse shaping. The preamble we use here for ranging consists of eight chirp symbols with all ones sequence. We generate power delay profiles based on IEEE 802.15.4a channel model 1 [14] which was measured within coverage from 7m to 20m in residential line of sight (LOS) environment.

#### A. Complexity Analysis

In order to allow the comparison between the MUSIC and the proposed algorithm, the computational complexity of both methods is investigated. Table II summarizes the basic operations for each algorithm where EVD and GE stand for eigenvalue decomposition and generalized eigenvalue decomposition. From Table II, the MUSIC algorithm requires:

- 1 autocorrelation matrix computation with $L$ multiplications
- 1 eigenvalue decomposition for $L \times L$ matrix
- 1 pseudo spectrum analysis with $L - L_p$ multiplications.

Besides, the proposed matrix pencil algorithm requires only one generalized eigenvalue decomposition. The generalized eigenvalue decomposition can be divided into full inversion and eigenvalue decomposition [12]. Accordingly, the proposed algorithm requires:

- 1 eigenvalue decomposition for $L_p \times L_p$ matrix
- 1 full inversion of $L_p \times L_p$ matrix.

Note that the number of multipath $L_p$ is much smaller than the number of frequency domain samples $L$. Even though the computational complexity depends on implementation, it is easy to verify that the proposed algorithm has lower complexity than the MUSIC algorithm.

#### B. Performance Analysis

The ranging error performance is presented in this subsection. Simulation is conducted to study the sensitivity of the proposed algorithm with respect to SNR. Simulating the MUSIC algorithm case, every step in the simulation is same as the proposed algorithm except TOA (or delay component) estimation parts for the fair comparison.

The performance of conventional super-resolution algorithm, MUSIC and the proposed scheme is depicted in Fig. 3 with the ranging estimation error and the corresponding standard deviation. Although the performance of MUSIC algorithm was expected higher than the proposed algorithm in general, the results show that the ranging accuracy of the proposed scheme is much better in the simulated indoor environment. The reason is that finding the peak in MUSIC pseudo spectrum involves with some error when the number of multipath increases [10]. Consequently, the performance degradation of ranging estimation using MUSIC algorithm is leaded. Meanwhile, the proposed scheme shows that the average ranging error is less than 70cm in most part of SNR range, and the standard deviation is also relatively small.

### TABLE III
COMPARISON FOR THE NUMBER OF OPERATIONS REQUIRED BY MUSIC AND THE PROPOSED MATRIX PENCIL ALGORITHMS

<table>
<thead>
<tr>
<th>Operation</th>
<th>Complexity</th>
<th>MUSIC</th>
<th>Matrix Pencil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication</td>
<td>$O(n^3)$</td>
<td>$2L - L_p$</td>
<td>-</td>
</tr>
<tr>
<td>EVD</td>
<td>$O(25n^3)$</td>
<td>$I_{(L \times L)}$</td>
<td>$I_{(L_p \times L_p)}$</td>
</tr>
<tr>
<td>Full inversion</td>
<td>$O(2n^3/3)$</td>
<td>-</td>
<td>$I_{(L_p \times L_p)}$</td>
</tr>
</tbody>
</table>

![Fig. 3. Average ranging errors with standard deviation using three different TOA estimation techniques](image-url)
which is within 1m. Therefore, the proposed algorithm might be suitable for the indoor applications.

V. CONCLUSION

In this paper, we proposed a novel ranging estimation technique employing chirp signalling. The proposed algorithm is composed of two-step signal processing: MMSE-based channel impulse response estimation and the channel delay tracking by matrix pencil. Through the MMSE-based channel impulse response estimation, we can obtain the CIR information. Also, the matrix pencil algorithm is applied to the measured channel frequency response in order to efficiently estimate TOA. Our results show that the proposed MMSE-based matrix pencil estimation technique can not only reduce the computational complexity, but also significantly improve ranging estimation performance in indoor environment. Hence, in many cases of interest, the proposed algorithm may be a promising technique for location based applications or services.

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REFERENCES

[13] P802.15.4a/D7, Approved Draft Amendment to IEEE Standard for Information technology-Telecommunications and information exchange between systems-PART 15.4: Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications for Low-Rate Wireless Personal Area Networks (LR-WPANs): Amendment to add alternate PHY (Amendment of IEEE Std 802.15.4), Jan. 2007