Efficient V-BLAST Detection Using Modified Fano Algorithm

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SUMMARY We propose a sub-optimal but computationally efficient Modified Fano Detection algorithm (MFD) for V-BLAST systems. This algorithm utilizes the QR decomposition of the channel matrix and the sequential detection scheme based on tree searching to find the optimal symbol sequence. For more reliable signal detection, the decoder is designed to move backward for the specified value at the end of the tree. This results in significant reduction of the complexity while the performance of MFD is comparable to that of ML detector.

key words: MIMO, V-BLAST, sphere decoding, Fano algorithm

1. Introduction

In an independent Rayleigh scattering environment, multiple antenna systems provide enormous increase in spectral efficiency compared to single antenna systems [1]. To take advantage of the multiple antenna systems, the V-BLAST (Vertical Bell Labs Layered Space-Time) architecture was proposed in [2]. For the detection of the vertically layered codes at a receiver, a variety of detection algorithms were introduced [2]–[4]. Although these schemes provide a reasonable performance at a feasible complexity, these performances may not be sufficient for systems with higher-order modulation due to the severe performance degradation caused by inter-antenna interference.

The Maximum Likelihood (ML) detector for Multi-Input Multi-Output (MIMO) systems [6] is known to be an optimal solution for the performance problem at the receiver. However, when a large number of antennas are used together with higher modulation constellations, ML detection is not feasible for real-time implementations. As an alternative, Sphere Decoding (SD) have been proposed in [7], [8] to avoid the exponential complexity of ML detector.

In this letter, we propose a sub-optimal but computationally efficient modified Fano detection algorithm for the V-BLAST system, which yields the performance close to that of ML with significant reduction in complexity. The Fano algorithm is one of the various sequential decoding algorithms for decoding convolutional codes [9], [10]. Sequential decoding can be viewed as a trial-and-error technique for searching out the correct path in the code tree. It also asymptotically achieves the same error probability as ML decoding without searching all possible states. As a similar approach, metric-guided (MG) algorithm applying basic steps of Fano algorithm has been proposed in [5]. The MG algorithm computes branch metric using the sum of differences between tolerance value and the actual detected noise energy. On the other hand, the proposed MFD uses Fano-like metric bias for the unequal length symbol sequences as a metric. The important characteristic of the proposed MFD is that the decoder is designed to move backward for the specified value at the end of the tree for more reliable signal detection.

This letter is organized as follows. In Sect. 2, we describe the system model. A V-BLAST detection scheme using modified fano algorithm is proposed in Sect. 3. We present the simulation results regarding performance and computational complexity in Sect. 4 and finally make conclusions in Sect. 5.

Notation: Throughout this letter, bold symbols denote matrices or vectors. \((\cdot)^T\), \((\cdot)^H\), and \((\cdot)^*\) denote transpose, Hermitian transpose, and conjugate, respectively. \(|.|\) denotes the Euclidean distance and \(L_e\) denotes the average number of real operations computed at the \(i\)-th tree level for MFD.

2. System Description

We consider a multiple antenna system with \(m\) transmit and \(n\) receive antennas, where usually \(n \geq m\). We assume ideal timing and symbol-synchronous receiver sampling, thus omit the time index for convenience. The baseband equivalent model of \(n\) dimentional received signal vector \(\mathbf{y} = [y_1 y_2 \cdots y_n]^T\) is given by

\[
\mathbf{y} = \mathbf{Hs} + \mathbf{w},
\]

where \(\mathbf{s} = [s_1 s_2 \cdots s_m]^T\) denotes the vector of transmitted symbols from \(M\)-PSK or \(M\)-QAM and \(\mathbf{w} = [w_1 w_2 \cdots w_n]^T\) denotes the zero-mean complex additive white Gaussian noise of variance \(\sigma_n^2\) while the average transmit power of each antenna is normalized to unity. The \(n \times m\) channel matrix \(\mathbf{H}\) contains uncorrelated complex Gaussian fading coefficients with unit variance and changes independently from frame to frame. We assume that the independent fading coefficients are perfectly known to the receiver.

Recently the V-BLAST detection approach using the QR decomposition of the channel matrix was introduced to avoid the intensive computational complexity of an ordered successive interference cancellation (OSIC) scheme [3]. Using QR decomposition of the channel matrix \(\mathbf{H}\), the suffi-
cient statistic for the transmit vector $s$ is obtained as

$$\tilde{y} = Rs + \eta.$$  \hspace{1cm} (2)

where the $n \times m$ matrix $Q$ is an unitary matrix and the $m \times m$ matrix $R$ is an upper triangular matrix. The statistical properties of the noise term $\eta = Q^H w$ remain unchanged.

3. Proposed V-BLAST Detection Algorithm

This section develops the computationally efficient V-BLAST detector using the modified Fano algorithm. As mentioned previously, the Fano algorithm was originally introduced as a sequential decoding scheme for convolutional codes to avoid the exponential complexity of a Viterbi algorithm (VA) caused by large constraint lengths. The decoder is based on generating hypotheses about the transmitted codeword sequence and computing a metric between these hypotheses and the received signal. It goes forward or backward repeatedly in tree searching, so that it finds the most likely path. In the following subsections, we present the details of the proposed detector.

3.1 Fano-Like Metric Bias

In order to employ the Fano algorithm in MIMO systems, it is necessary to adjust a metric on symbol based operation taking into account the lengths of the different paths being compared. This metric throughout this letter is called a Fano-like metric bias [8]. Like the Fano metric for the different path length in sequential sequence detection, it provides a criterion that compares the unequal length symbol sequences. Hence, the branch metrics in higher levels have a larger bias than those in lower levels. It reflects that they are much closer to the end of the tree and thus more likely to be part of the best path. The best path implies the one with the smallest Fano-like metric. From Eq. (1), the average value of the smallest path is given by [8]

$$E(||y - Hs||^2) = E(||w||^2) = n \sigma^2_w.$$  \hspace{1cm} (3)

Therefore, it is reasonable to choose $\alpha \sigma^2_w$ as the Fano-like metric bias for computing the branch metric at each tree level, where $0 \leq \alpha \leq 1$. The value of $\alpha$ can be adjusted empirically by simulations. More specifically, $\alpha = 0$ implies that there would be no Fano-like metric bias while $\alpha = 1$ implies that there would be the lowest complexity with severe degradation in the bit error rate (BER) performance. Hence, $\alpha$ is simply set to be 0.5 in our simulations as a performance trade-off. Moreover, the squared distance, which will be shown in Eq. (6), is proportional to $r_{k,k}$ for the level $k$. The Fano-like metric bias for the tree level $k$ can be thus defined as follows

$$F_k = F_{k-1} + \alpha \sigma^2_{w} r_{k-1,k-1}, \quad (k = 2, 3, \cdots, m),$$  \hspace{1cm} (4)

where $F_1$ is defined as zero and $r_{k-1,k-1}$ represents $(k-1, k-1)$ component in the upper triangular matrix $R$.

3.2 Modified Fano V-BLAST Detection Algorithm Description

Now we consider a $M$-ary tree structure for the modified Fano V-BLAST detector. Figure 1 illustrates a simple binary tree structure as an example. The decoding process of the modified version begins at the root node and terminates if one of two parameters (i.e., $N_{bm}$ and $N_{mmm}$) reaches to the specified value. The original Fano decoding process, on the other hand, terminates only when the decoder reaches to the level $m$ of the tree. This level is equal to the number of the transmit antennas. To improve the decoding performance, we introduce two parameters, $N_{bm}$ and $N_{mmm}$, which indicate the number of the backward movement by force in the level $m+1$ and the repeated number of the same accumulated minimum metric at the level $m$, respectively. From the root node with a threshold $T = 0$ and a metric value $MV = 0$, the decoder looks forward to finding the best of the $M^k$ succeeding nodes at the tree level $k$. The best node is determined by the biased cumulative path metric which is based on the Euclidean distance which will be defined in Eq. (6).

Due to the upper triangular structure of $R$ in Eq. (2), the $k$-th element of $\tilde{y}$ is rewritten as

$$\tilde{y}_k = r_{k,k} s_k + \sum_{i=k+1}^{m} r_{k,i} s_i + \eta_k.$$  \hspace{1cm} (5)

Using the Fano-like metric bias in Eq. (4), the $i$-th biased branch metric at the tree level $k$ is computed as
where $c_i$ represents the element of the signal constellation. Among all the branch metrics calculated at each level, the node having the minimum branch metric corresponds to the best one. Especially we assume that all the branch metrics are zeros if the decoder reaches at the level $m + 1$.

At each node in the tree, the decoder makes a decision whether it moves forward or backward. If the forward metric $M_F$ [10] is less than or equal to the threshold $T$, the decoder moves forward to the best of the $M_k$ succeeding nodes, otherwise any of several events may occur, depending on the value of the backward metric $M_B$. Along the various decoding steps, the threshold is tightened or adjusted by a non-zero integer multiple (i.e., $\Delta = 3$ in our case). This prevents the decoder from getting stuck in an infinite loop. Once the decoder reaches at the end of tree level $k = m$, it computes the following metric in (6) during the limited $N_{bm}$ (i.e., 200 or 400 in our case),

$$J(s) = -2\text{Re}[s^H H^H y] + \text{Re}[s^H H^H H s].$$

The backward movements provide various symbol sequences as candidates and then the decoder computes the metrics for these symbol sequences. Among them, the decoder finds the sub-optimal symbol sequence which has the minimum metric calculated in Eq. (7). During this procedure, the same minimum value of the metric would be repeated. Finally, the decoder terminates the decoding process upon this parameter, $N_{rnm}$, or the parameter, $N_{bm}$. Figure 2 presents the complete flowchart of the MFD. As shown in this flowchart, the MFD is designed to move backward for the specified value at the tree level $m + 1$. For this procedure, the value of $T$ must be changed twice successively: it is changed to sufficiently small value (in our case, $T = -100$) for the decoder to look back, and then it is changed to sufficiently large value (in our case, $T = 100$) once again for the decoder to move backward.

4. Simulation Results

We investigate the performance of our proposed algorithm using the V-BLAST system with $m = 4$ transmit and $n = 4$ receive antennas employing uncoded 16QAM modulation at Rayleigh fading channels. We specified the case of $N_{bm} = 200$ or 400 to obtain the reasonable BER performance and considered $N_{rnm} = 2$ and $N_{rnm} = 4$. These parameters can be adjusted depending on the desired performance and complexity. The average bit energy to noise power ratio ($E_b/N_o$) is defined as the SNR at the receiver normalized by the number of bits per symbol, thus $E_b/N_o = m / (\log_2(M) \sigma_w^2)$ is used.

The proposed V-BLAST detector is compared with ML and MMSE-OSIC detectors using the Monte Carlo simulations in terms of the average bit error rates (BER) performance and the corresponding computational complexity. Especially, we evaluate the computational effort of each algorithm by counting the real operations including multiplication, addition, and division. We also consider all the steps required to detect the transmitted signal at the receiver for each algorithm respectively. For this evaluation, each complex operation is converted into real operation to get a better idea of complexity. For instance, one complex multiplication is equivalent to three real multiplications and five real additions. Table 1 shows the complexity evaluation of the proposed algorithm, compared with the conventional
Table 1 Complexity evaluation of each algorithm [11].

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Complexity Formula</th>
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<tbody>
<tr>
<td>ML</td>
<td>$(30m^2 + 8m^2 + 6m - 3n)m^m$</td>
</tr>
<tr>
<td>MMSE-OSIC</td>
<td>$\frac{25}{2}m^4 + 9m^3 + 20m^2n - \frac{1}{2}m^2 + 2m^2n + 20nm$ $- 10n + \frac{3}{2}$</td>
</tr>
<tr>
<td>MFD</td>
<td>$10mn^2 - n^2 + 2nm + \sum_{i=1}^{m} L_i$</td>
</tr>
</tbody>
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Fig. 3 Performance comparison between MFD and other schemes with $m = 4$ and $n = 4$ antennas, uncoded 16QAM symbols, 16 bps/Hz.

In Fig. 3, we observe that the BER performance of proposed algorithm approaches that of ML detection as the repeated number of minimum metric ($N_{\text{mm}}$) increases. However, MFD with $N_{\text{mm}} = 4$ is still inferior to ML by approximate 1 dB, whereas MFD outperforms MMSE-OSIC by 6 dB even in the case of $N_{\text{mm}} = 2$. This is because MMSE-OSIC at high SNR suffers error propagation and the proposed MFD is fundamentally based on the ML principle.

As explained in Sect. 3, the proposed decoder moves forward or backward repeatedly until it finds the optimal symbol sequence. To show the tendency of computational efforts of our scheme according to various $N_{\text{mm}}$ values together with other schemes, the average computational effort for the MFD and the conventional V-BLAST detectors is plotted in Fig. 4. From this result, we observe that the computational effort of the MFD is much less than that of ML as the SNR increase, however the MFD still has more complexity than MMSE-OSIC.

From the simulation results, we observe the trade-off between the BER performance and the decoder complexity. For more reliable signal detection, we add two parameters, $N_{\text{bm}}$ and $N_{\text{mm}}$, to our approach. As $N_{\text{bm}}$ and $N_{\text{mm}}$ increase, the BER performance of MFD is closer to that of ML. However, the improvement is achieved at the cost of the increase of the computational complexity. Nevertheless, from Fig. 4, it is clear that the proposed MFD has much less computational complexity compared to the ML detection with highly favorable performance.

5. Conclusions

We proposed a sub-optimal, reduced complexity MFD algorithm well-suited to MIMO systems with layered space-time architecture. The MFD is based on sequential sequence detection scheme using tree searching with backward movement. The comparisons with the existing algorithms exhibit that the performance of MFD algorithm is significantly improved compared with the conventional V-BLAST detector (i.e., MMSE-OSIC). Also the computational complexity of our approach is noticeably reduced as against that of ML detector. For example, the computational effort of the MFD is about 650 times less than that of ML at high SNR region even when $N_{\text{bm}} = 400$ and $N_{\text{mm}} = 4$ are used. Therefore, the proposed algorithm can be a practical alternative scheme for the V-BLAST system. We also expect that our proposed algorithm can be applied to other MIMO schemes in high traffic demanding environments.

Acknowledgments

This research was supported by University IT Research Center Project.

References