Frame Synchronization Exploiting Cauchy Distribution in DMT-Based xDSL Modems

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1. Introduction

In recent years, the digital subscriber loop (DSL) has grown fast as a dominant last-mile technology for the high-speed Internet access. Unlike its competitor cable modem that requires new cable line installation, DSL uses already existing twisted pair telephone lines. DMT modulation technique was selected as the standard for ADSL [2] and VDSL modems [3].

In general, sampling time generation is known to be a critical part in DMT-based xDSL modems. There are two methods of sampling time generation at the receiver. In synchronous sampling DMT systems, voltage controlled oscillator (VCO) is used to generate the sampling time. When there exists timing error between the transmitter and receiver clocks, timing error is detected in the digital domain and the error is fed back to the analog components VCO to adjust the sampling time. In non-synchronous sampling systems, a free running crystal is used to generate the sampling time without any feedback from the digital domain. Therefore, the timing error has to be detected and corrected in the digital domain [4]. This paper focuses on the non-synchronization sampling scheme.

It was shown before that sampling time error between transmitter and receiver clocks causes a severe bit error rate (BER) performance degradation of DMT/OFDM systems [5]. There are two timing error cases, sampling phase offset (SPO) and sampling frequency offset (SFO) at the sampling device of receiver. In general, compensation of SPO is easy using the delay-rotor property since every sample within each frame experiences the same amount of sampling time error while SFO makes the compensation difficult. It is proposed that a timing error correction scheme in digital domain in [4], assuming ideal sampling time error estimation and frame synchronization.

In reality, however, we need to detect the first sample time in each frame (frame synchronization) because the transmitted signals arrive at the receiver after an unknown channel delay. In DMT system, the failure of frame synchronization results in inter-symbol interference (ISI) and inter-channel interference (ICI). Therefore, the frame synchronization must be performed before the demodulating FFT at the receiver. It was also shown that the frame synchronization is very important to achieve a good performance of the DMT-based VDSL modems [6], [7].

The SFO makes the frame synchronization difficult by linearly increasing or decreasing sampling time error of samples even inside a frame. Therefore, we need to consider both SPO and SFO for the delay estimation. This problem gets more serious as the symbol length increases (larger FFT size) as in the recently proposed scheme for VDSL [3].

A maximum likelihood (ML) algorithm was proposed for the frame synchronization maximizing a non-data-aided (NDA) likelihood function of the received signal [1]. The function is based on the correlation between two sequences separated by 2N samples, where 2N is the FFT size. The ML method takes advantage of M correlators and an averaging device. The ML algorithm tries to find the channel delay $D$ that maximizes the likelihood function. A large number of correlators used in the ML algorithm, however, can cause a significant demodulation delay as well as high computational complexity. Furthermore, the ML algorithm that uses rather many correlators does not work properly when SFO is significant because of non-uniform sampling time error of samples even within a frame.

In this paper, a new computationally efficient frame synchronization method which shows relatively reliable performance even with a significant SFO is proposed. The proposed approach is based on the observation that the normalized correlation between two sequences separated by 2N samples is Cauchy distributed with different center, depending on whether the two corresponding sequences are corre-
lated or not. The proposed method relieves the demodulation delay problem of the previous approach, because only a small number of correlators are involved in the proposed approach. Therefore, ADSL/VDSL modems may reduce both the demodulation delay and computational complexity by using the proposed frame synchronization method. In Sect. 2, we review the previous approach briefly and then propose a new approach. In Sect. 3, simulations results are provided to demonstrate the effectiveness of the proposed approach, comparing with the previous approach. Conclusions are drawn in Sect. 4.

2. Frame Synchronization

2.1 A Previous Approach

A maximum likelihood (ML) algorithm was proposed for the frame synchronization maximizing a non-data-aided (NDA) likelihood function of the received signal. An NDA likelihood function was derived over an AWGN channel in [8] and its approximated form [1] is

$$F(D) = \sum_{m=0}^{M-1} \sum_{n=0}^{v-1} r((n + m(2N + v) + D)T) \times r^*(((n + m(2N + v) + 2N + D)T)$$

where $D$ is the channel delay which consists of integer part $i$ and fractional part $e$. Equation (1) is based on the correlation between two sequences separated by 2N samples. Figure 1 depicts the structure of the NDA delay estimation when $M = 3$. To estimate the delay of a frame, $M$ correlators are used to detect $D$ that maximizes $F(D)$. When the SFO is small, this estimation method works efficiently with large $M$. When the fractional part $e$ is fixed, an integer value which maximizes the function corresponds to an estimate of $D$ with an error of the fixed fractional part $e$ [1].

However, since the frame synchronization needs to precede the demodulation, the demodulation delay grows larger as the number of correlators increases. Thus, a significant demodulation delay can be caused by a large $M$ in the previous approach. Moreover, a larger number of correlators of the previous approach make it computationally expensive. Furthermore, this method shows severe performance degradation with the significant SFO. Since the SPO linearly increases/decreases by the accumulated SFO, a significant SFO results in severe SPO difference between the first sample and the last sample of each frame. If one symbol consists of $2N + v$ samples and the normalized SFO is $\Delta f$, then the n-th sample of k-th symbol experiences PO of $(n + k(2N + v)\Delta f)$ compared while the first sample of the first DMT symbol experiences zero phase offset.

The next section proposes a computationally efficient approach that mitigates both the significant SFO impact on frame synchronization performance and the delay problem.

2.2 A Proposed Approach

This section proposes a new likelihood function for the frame synchronization under consideration. Since the previous approach needs relatively large data samples which induce a long demodulation delay and high complexity for the frame synchronization, the following method proposes to use less data samples for a short delay. Using a new defined indication function, $\mathbf{1}(\cdot)$, and normalized correlation function, $q(n)$, a new likelihood function is proposed as

$$G(D) = \sum_{m=0}^{M-1} \sum_{n=0}^{v-1} \mathbf{1}(r((n + m(2N + v) + D)T)) \in CP) \times q(n)$$

where

$$q(n) = \begin{cases} \frac{r^*((n+2N+D)T)}{r((n+2N+D)T)} & \text{if } |\frac{r^*((n+2N+D)T)}{r((n+2N+D)T)} - 1| < |\frac{r^*(n(2N+D)T)}{r((n+2N+D)T)} - 1| \\ \frac{r(n+2N+D)T)}{r((n+2N+D)T)} & \text{otherwise} \end{cases}$$

and

$$\mathbf{1}(r((n + D)T)) \in CP) = \begin{cases} 1 & \text{if } 1 - \tau < q(n) < 1 + \tau \\ 0 & \text{otherwise.} \end{cases}$$

Equation (2) is the normalized correlation between two sequences separated by 2N samples. If a sample inside the correlator window whose length is $v$ samples corresponds to symbol $0$, and $2N$ symbols (CP) before and after symbol $0$.
to the CP segment, the normalized correlation \( q(n) \) of the samples is added as shown in Eq. (2). Thus, the increment and decrement of the normalized correlation function are also normalized to evoke maximum function value with less samples whereas the previous approach uses relatively more samples. Note that whether each sample inside the correlator window corresponds to CP segment or not is decided by the indicator function. The indicator function takes \( q(n) \) and decides that the corresponding sample is inside the CP segment if \( q(n) \) satisfies the condition in Eq. (4). The parameter \( \tau \) will be addressed later. Therefore, the \( q(n) \) of a sample is not added when the sample is outside of the CP segment.

We now address the indicator function. The indicator function is based on the observation of normalized correlation \( q(n) \). The function \( q(n) \) is the ratio of two complex Gaussian random variables with zero mean and a fixed variance. As proved in APPENDIX, \( q(n) \) is a Cauchy random variable with different centers depending on the correlation of the two Gaussian random variables. Figure 2 shows the distributions with different correlation between the two random variables. As shown in Fig. 2, the PDF of \( z \) moves from the left dashed-line distribution to the right solid-line distribution as \( y_{xy} \) gets closer to 1 and the \( q(n) \) turns out to be Cauchy distributed centered approximately at 1 when the corresponding sample \( r((n + D)T) \) is inside the CP. When the sample \( r((n + D)T) \) is outside the CP, however, the center of \( q(n) \) comes close to 0. Inspired by this observation, we propose the indicator function. If the normalized correlation \( q(n) \) is of a sample corresponding a CP segment, the value \( q(n) \) is expected to be Cauchy random variable centered at 1. Therefore, if the value \( q(n) \) satisfies the following condition

\[
1 - \tau < q(n) < 1 + \tau
\]  

(5)
as shown in Fig. 2, the indicator function indicates that the corresponding sample is inside the CP segment. When the sample comes from a CP, however, the center of distribution does not exist exactly centered at 1 but around 1 due to the effects of noise and a raised cosine filter [9].

With the indicator and normalized correlation function, the proposed method uses a small number of correlators because the use of a large number of correlators worsens the frame synchronization performance in the presence of a significant SFO. Since the new approach uses only \( M' \) symbols which is much less than \( M \) symbols of the previous approach, the demodulation delay as well as computational complexity is significantly reduced.

### 3. Simulations

The effectiveness of the proposed approach is investigated with the simulation experiments. For performance comparison, both of conventional and proposed methods are simulated under the same condition. A 30 dB SNR is assumed in the time domain and moderate SFO of 0.25 is assumed. As for SFO, 100 ppm and 200 ppm are assumed for a typical condition and a severe condition, respectively. A roll off factor of 0.03 is used for the transmitter and receiver raised cosine filters. The FFT size of 512 and CP length of 32 are adopted in the simulation according to the ADSL standard [2]. Although bit allocation (adaptive modulation) is used in practical systems, we use only 16-QAM for symbol mapping because there is no difference from the frame synchronization perspective. In the conventional method, seven symbols and corresponding seven correlators (\( M = 7 \)) are used for the frame synchronization. In the proposed method, only four correlators (\( M' = 4 \)) are used.

The delay estimation performance of the conventional and the proposed methods with different SFOs is shown in Fig. 3 and Fig. 4. The estimation error is defined as \( |D - \hat{D}| \), where \( D \) is the true channel delay and \( \hat{D} \) is the estimated channel delay by the two methods. As can be observed in Fig. 3 and Fig. 4, the estimation performance of the proposed method that involves only four correlators is very similar with the performance of the conventional when SFO is 100 ppm. Moreover, the proposed method shows relatively reliable performance than the conventional method which shows severe performance degradation when SFO is significant. The probability of exact frame synchronization of the proposed method is 0.55 while that of the conventional method is 0.1 when SFO is 200 ppm. Therefore, we can conclude that the proposed frame synchronization scheme

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**Fig. 2** Cauchy PDF when \( y_{xy} = 0 \) and \( y_{xy} = 0.99 \).

**Fig. 3** Histogram of error of the conventional and the proposed synchronization methods with SFO = 100 ppm.
is better than the previous method in terms of estimation performance, computational complexity, and demodulation delay.

4. Conclusion

We propose a novel frame synchronization method for DMT-based xDSL systems in the presence of timing error. The proposed method is based on the normalized correlation which is shown to be a Cauchy random variable. Since the proposed method involves less number of correlators, computational complexity as well as demodulation delay is reduced. Furthermore, the estimation performance degradation of the conventional method is mitigated by the proposed method when SFO is significant. The proposed technique would be applied to ADSL/VDSL systems for the low cost implementation even with better performance. Simulations were conducted, adopting ADSL standard parameters and showed the superior performance of the proposed frame synchronization method.

Acknowledgments

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References


Appendix

We prove that the normalized correlation coefficient which is given below is a Cauchy random variable. We assume the following

\[ z = \frac{x}{y} \] (A-1)

If \( x \) and \( y \) are jointly normal random variables with zero mean, their joint probability density function (PDF) is given as [10]

\[ f_{xy}(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\gamma_{xy}^2}} \exp\left\{ \frac{1}{2(1-\gamma_{xy})} \left( \frac{x}{\sigma_x} - \frac{2\gamma_{xy}y}{\sigma_x\sigma_y} + \frac{y}{\sigma_y} \right) \right\} \] (A-2)

where \( \gamma_{xy} \) is a correlation coefficient. From Eq. (A-2), the PDF of can be derived as

\[ f_z(z) = \frac{1}{\pi\sigma^2(z-\gamma_{xy})^2 + \sigma^2(1-\gamma_{xy})^2} \] (A-3)

The equation (A-3) represents a Cauchy distribution centered at \( \frac{\sigma_x}{\sigma_y} \) and \( \gamma_{xy} \) is defined as follows

\[ \gamma_{xy} = \frac{E[xy]}{\sqrt{E[x^2]E[y^2]}} \] (A-4)

If \( \sigma_x = \sigma_y \) as is the case under consideration in the paper, the PDF of \( z \) can also be expressed as

\[ f_z(z) = \frac{1}{\pi\sqrt{1-\gamma_{xy}^2}} \sqrt{1-\gamma_{xy}^2} \] (A-5)

Equation (A-5) represents a Cauchy distribution centered at \( \gamma_{xy} \).