Iterative Near-optimal Decoder for STBC from CIOD over Time-Varying Channel

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Abstract—Space-time block code from coordinate interleaved orthogonal design (STBC-CIOD) is considered as an effective transmit diversity scheme for achieving full-rate and full-diversity with low decoding complexity. The simple maximum likelihood (SML) decoder for STBC-CIOD is known to be an optimal decoder in time-invariant channel. However, the performance of the SML decoder is inevitably degraded when the channel becomes time-varying. In this paper, we propose a computationally efficient near-optimal decoding algorithm for STBC-CIOD to mitigate the interference caused by time-selectivity of the channel. The algorithm establishes partial candidate sequences and iteratively finds the best sequence among the sequences through metric comparison. Simulation results show that the proposed decoder outperforms the SML decoder and achieves near-optimal performance with significantly low complexity.

I. INTRODUCTION

Multiple antenna systems incorporating with a proper signal processing provide outstanding high spectral efficiency in rich scattering environments compared to single antenna systems [1]. Space-time block code (STBC) for multiple-input multiple-output (MIMO) systems, that is firstly introduced in [2], improves system performance under quasi-static channel assumption. STBC for two transmit antennas is known to have orthogonal design and to achieve full-rate as well. However, when the number of transmit antennas is greater than two, the bandwidth utilization of the STBC from orthogonal design becomes inefficient, since the maximum symbol transmission rate is lower than one [3],[4]. To resolve this bandwidth inefficiency, quasi-orthogonal STBC (QO-STBC) is introduced in [5] with the loss of orthogonality of the code that is incapable of achieving full-diversity gain. For a full-diversity and full-rate (FDFR) code design, constellation rotation based QO-STBC (CR based QO-STBC) and STBC from co-ordinate interleaved orthogonal designs (STBC-CIOD) are proposed in [6]-[8]. The STBC-CIOD is designed to have low decoding complexity over CR based QO-STBC, since encoded signals are single-symbol decodable with simple maximum likelihood (SML) decoder under quasi-static assumption [9]. Note that SML decoder consists of linear combining and a scalar ML detection applied to individual data symbol.

However, when the quasi-static channel condition is not satisfied, unexpected interference among nearby antennas arises and results in severe performance degradation. As a result, SML designed for quasi-static fading channel is no longer optimal for STBC-CIOD decoder in the time-varying environment. A joint-maximum likelihood (JML) decoder[10] can be employed to resolve the interference problem. The JML, however, is impractical because of its high computational complexity. Thus, considering size and power limitation of the mobile station, it is of importance to have a decoder with reasonable complexity that overcomes the interference due to time-varying fading.

In this paper, we aim to propose a low complexity decoding scheme for CIOD that reduces the unexpected interference. The proposed scheme begins with QR decomposition of the channel matrix, then the candidate list of the half part of the signals are generated by the combination of the most likely symbols corresponding to two signals. From the top of the partial candidate list, the metrics of the remaining half part of the signals are iteratively compared and an optimal sequence that has minimum sum metric is decided. In order to reduce computation complexity, an early termination condition is employed. Simulation results demonstrate that the proposed algorithm achieves the performance close to that of JML over time-varying channel.

This paper is organized as follows. Section II describes the model of STBC-CIOD and of special time-varying channel. In section III, we present the structure of proposed decoder in comparison with that of the SML decoder. The simulation results for performance and complexity are provided in section IV. Finally, we make a conclusion in section V.

Notation: Throughout this paper, bold symbols denote matrices or vectors. ($\bullet)^T$ and ($\bullet)^H$ denote transpose and Hermitian transpose, respectively.

II. SYSTEM DESCRIPTION

A. System Model

The Fig.1 depicts the block diagram of STBC-CIOD system with four transmit antennas ($n_T = 4$) and one receive antenna ($n_R = 1$). In the system considered in this paper, a rotated constellation [13] is not only employed for the maximum coding gain, but also it can acquire the diversity gain through interleaved symbols.
where $\tilde{y}$ is more accurate in realizing Rayleigh fading channel than due to several factors such as Doppler spread generated from assumption. However, the variation of the channel is unavoidable to recover the original symbols, by transmit symbol period, the received signal can be represented as

$$\tilde{y} = \tilde{S}h + n,$$

where $h$ denotes the channel vector, and $n$ is a noise vector. For the convenience of representation of the decoding process, the received signal is rewritten as

$$\tilde{y} = H\tilde{S} + \tilde{n},$$

where

$$H = \begin{bmatrix} h_1 & h_2 & 0 & 0 \\ h_2^* & -h_1^* & 0 & 0 \\ 0 & 0 & h_3 & h_4 \\ 0 & 0 & h_4^* & -h_3^* \end{bmatrix},$$

$$\tilde{y} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \\ \tilde{y}_4 \end{bmatrix}, \quad \tilde{S} = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \end{bmatrix}, \quad \text{and} \quad \tilde{n} = \begin{bmatrix} n_1 \\ n_2^* \\ n_3 \\ n_4^* \end{bmatrix}.$$

Since the transmitted signals are coordinate interleaved at transmitter, the received signals, $\tilde{y}$, need to be deinterleaved to recover the original symbols, $y$.

### B. Channel Model

The STBC-CIOD is designed under quasi-static channel assumption. However, the variation of the channel is unavoidable due to several factors such as Doppler spread generated from the mobility of the station. To represent this time-varying characteristic of the channel, a Markov model is used. In [11], it is verified that the first order Markov channel model is more accurate in realizing Rayleigh fading channel than Jakes’ model which is generally employed to simulate the fading channel. A particular channel model-based the first order Markov process [12] is given by

$$h(n) = ah(n-1) + g(n), \quad n = 2, \ldots, nr,$$

where $h(n)$ is the complex channel gain with $i$-th transmit antenna at the time slot $n$ with zero mean and the variance of $\sigma_h^2 = 1$. Also, $g(n)$ denotes an additive white Gaussian random variable with zero mean and the variance of $\sigma_g^2$. Then, $\alpha$ works as a channel offset factor ($0 < \alpha < 1$) which depends on Doppler shift of the channel. For example, $\alpha = 1$ implies that the channel model is quasi-static. Conversely, if $\alpha = 0$, then the channel components become independent at each time slot. To normalize $h(n)$, the following condition needs to be satisfied:

$$\sigma_h^2 = |\alpha|^2 + \sigma_g^2 = 1.$$

Based on this time varying model, the channel matrix for the CIOD can be expressed as

$$H = \begin{bmatrix} h_1(1) & h_2(1) & 0 & 0 \\ h_2(2)^* & -h_1(2) & 0 & 0 \\ 0 & 0 & h_3(3) & h_4(3) \\ 0 & 0 & h_4(4)^* & -h_3(4) \end{bmatrix}.$$

### III. PROPOSED LOW COMPLEXITY DECODING ALGORITHM

In this section, we introduce the proposed decoding algorithm. The conventional SML decoder shows the ML performance with low computational effort under quasi-static channel assumption. However, the loss of orthogonality due to time-variant channel coefficients results in performance degradation of the SML decoder. Therefore, we propose a novel decoding algorithm to relieve the impairment from the time-selective channel. In comparison with the conventional SML decoder, the proposed algorithm achieves remarkable improved performance with a reasonable additional computational cost.

#### A. Conventional Detection Algorithms

In this subsection, we present how SML and JML decoders work. The SML decoder is a simple detection algorithm that consists of matched-filtering and single-symbol decoding. After matched-filtering at the receiver, the received signal is represented as

$$\tilde{v} = H^T\tilde{y}.$$

The SML decoder only considers diagonal values of $H^T H$ and the off-diagonal terms are not counted in decoding. The diagonal values of $H^T H$ is $D = |h_1(1)|^2 + |h_2(2)|^2 |h_2(1)|^2 + |h_1(2)|^2 + |h_3(3)|^2 + |h_4(4)|^2 + |h_4(3)|^2 + |h_3(4)|^2$.

Since the transmitted signals are coordinate interleaved at the transmitter, the received signals should be deinterleaved to
As the first step of the proposed algorithm, QR decomposition of the channel matrix \( \mathbf{H} \) is obtained as \( \mathbf{H} = \mathbf{QR} \) where \( \mathbf{Q} \) is the 4-by-4 unitary matrix and \( \mathbf{R} \) is the 4-by-4 upper triangular matrix. By multiplying \( \mathbf{Q}^H \) to both sides, Eq. (3) is rewritten as

\[
\bar{\mathbf{v}} = \mathbf{Q}^H \mathbf{v} = \mathbf{R} \hat{s} + \mathbf{Q}^H \mathbf{n}.
\]  

(11)

Note that the \( \mathbf{R} \) matrix has a special form as

\[
\mathbf{R} = \begin{bmatrix}
1.1 & 1.2 & 0 & 0 \\
0 & 2.2 & 0 & 0 \\
0 & 0 & 3.3 & 3.4 \\
0 & 0 & 0 & 4.4
\end{bmatrix}
\]  

(12)

Since upper-right four elements of the upper triangular part of the \( \mathbf{R} \) are zero, signals that constitute the first orthogonal code (i.e., \( \hat{s}_1 \) and \( \hat{s}_2 \)) and signals that constitute the second orthogonal code (i.e. \( \hat{s}_3 \) and \( \hat{s}_4 \)) can be detected separately. From this observation, it is possible to construct a detector with the following three-step detection process. The ordered list of the candidate sequence of \( \hat{s}_2 \) and \( \hat{s}_4 \) is established and the remaining signals, \( \hat{s}_1 \) and \( \hat{s}_3 \), are iteratively searched based on the partial candidate sequences. Before the iterative algorithm, the signal should be de-interleaved like

\[
\tilde{\mathbf{v}}_i = \tilde{\mathbf{v}}_{iI} + \tilde{\mathbf{v}}_{iQ},
\]  

(13)

where \( \tilde{\mathbf{v}}_{iI} \) and \( \tilde{\mathbf{v}}_{iQ} \) are the in-phase and the quadrature-phase components of interleaved \( \mathbf{v}_i \), respectively. The specified detection procedures are described in the following steps.

**Step 1 (initialization):** In this step, partial candidate sequences of \( \mathbf{v}_2 \) and \( \mathbf{v}_3 \) are established by the combination of the most likely signals corresponding to the two signals. To construct the partial candidate list, the metric of the \( i \)-th constellation point for \( i = 1, \ldots, M \) is calculated by using the ML criterion as follows:

\[
m_{2i} = |\mathbf{v}_2 - r_{2i} \hat{s}_{iL}|^2 + |\mathbf{v}_2 - r_{2i} \hat{s}_{iQ}|^2,
\]  

(14)

\[
m_{4i} = |\mathbf{v}_4 - r_{4i} \hat{s}_{iL}|^2 + |\mathbf{v}_4 - r_{4i} \hat{s}_{iQ}|^2.
\]  

(15)

From Eq. (14), all the possible \( M^2 \) combinations of \( m_{2i} \) and \( m_{4i} \) are computed and ordered. The ordered cumulative metric represents \( \mathbf{m} = [\tilde{m}_1, \ldots, \tilde{m}_{M}]^T \) and the corresponding ordered list is \( \mathbf{s} = [\tilde{s}_1, \ldots, \tilde{s}_M]^T \) where \( \tilde{s}_i = [\tilde{s}_{iL}, \tilde{s}_{iQ}]^T \) and \( \tilde{m}_i = m_{2i} + m_{4i} \), and \( p, q \in \{1, \ldots, M\} \). The first sequence of the candidate list, \( \tilde{s}_1 \), has the smallest metric \( \tilde{m}_1 \).

**Step 2 (iterative searching):** According to the sequence obtained from prior step, the remained signals succeeding each candidate sequence are iteratively searched until an optimal full sequence is selected within the ordered list.

When the candidate sequence is assumed to be \( \tilde{s}_i \), the metrics of the first and third signals are computed as follows:

\[
m_{1i} = |\mathbf{v}_1 - a_{1i} \tilde{s}_{iL}|^2 + |\mathbf{v}_1 - a_{1i} \tilde{s}_{iQ}|^2,
\]  

(16)

\[
m_{3i} = |\mathbf{v}_3 - a_{3i} \tilde{s}_{iL}|^2 + |\mathbf{v}_3 - a_{3i} \tilde{s}_{iQ}|^2.
\]  

(17)

As the second step of the algorithm, the JML detection is a sort of a full searching detection algorithm. The JML detection is constructed from partial candidate sequences and corresponding metrics for \( \hat{s}_j \) and \( \hat{s}_k \). The JML detection is a sort of a full searching detection algorithm. The JML detection is constructed from partial candidate sequences and corresponding metrics for \( \hat{s}_j \) and \( \hat{s}_k \).
where

\[ a_{1,T} = \text{Re}(r_{1,2})\hat{s}^T_{2,T} - \text{Im}(r_{1,2})\hat{s}^T_{4,Q} \]
\[ a_{1,Q} = \text{Im}(r_{3,4})\hat{s}^Q_{4,Q} - \text{Re}(r_{3,4})\hat{s}^Q_{2,Q} \]
\[ a_{3,T} = \text{Re}(r_{3,4})\hat{s}^T_{4,T} - \text{Im}(r_{3,4})\hat{s}^T_{2,Q} \]
\[ a_{3,Q} = \text{Im}(r_{1,2})\hat{s}^Q_{2,Q} - \text{Re}(r_{1,2})\hat{s}^Q_{4,Q} \]

\( a_1 \) and \( a_3 \) denote interference from \( \hat{s}_2 \) and \( \hat{s}_4 \) to \( \hat{s}_1 \) and \( \hat{s}_3 \), respectively, which should be removed. Note that the detected signal for \( \hat{s}_2 \) and \( \hat{s}_4 \), \( \hat{s}_2^T \) and \( \hat{s}_2^Q \), are interleaved for the interference cancellation, considering the code design for CIOD.

Let the detected full sequence with \( \hat{s}_1 \) denotes \( s_{1}^{\text{full}} = [\hat{s}_1^T, \hat{s}_2^T, \hat{s}_3^T, \hat{s}_4^T] \), where \( r, s \in \{1, \cdots , M\} \) and each corresponding metric of \( \hat{s}_1^T \) and \( \hat{s}_1^Q \) has the smallest value of \( m_1^T \) and \( m_1^Q \), respectively. Also, the cumulative metric of \( s_{1}^{\text{full}} \) is calculated as \( m_{1}^{\text{full}} = m_1^T + m_1^Q \). When the current full sequence \( m_{1}^{\text{full}} \) is smaller than the threshold \( \eta \), choose \( s_{1}^{\text{full}} \) as a new optimal full sequence and replace \( \eta \) by \( m_{1}^{\text{full}} \). If \( m_{1}^{\text{full}} \) is greater than \( \eta \), it signifies that previous detected sequence is an optimal, go next step.

**Step 3 (termination):** To complete the algorithm with the most likely sequence, \( m^{\text{opt}} \) is compared with \( \eta \). The larger \( m^{\text{opt}} \) than \( \eta \) implies that a current cumulative metric, \( s_{\text{opt}}^{\text{full}} \), might not be an optimal value. Thus, the algorithm implements **Step 2** again with \( s_{\text{opt}}^{\text{full}} \) to find more likely sequence. If not, \( s_{\text{opt}}^{\text{full}} \) is decided as an optimal sequence since \( \eta \) is absolutely smaller than the metrics of all the remaining candidate sequences, \( m^{\text{opt}+1}, \cdots , m^{\text{opt}+T} \). Because the algorithm is able to terminate early without unnecessary efforts, it is more effective than JML in computational complexity. When all the candidate sequences are examined, the algorithm is also terminated.

### IV. SIMULATION RESULTS

In this section, Monte Carlo simulation results are presented using the average bit error rate (BER) with respect to SNR. Corresponding complexity is also compared to verify the effectiveness of the proposed algorithm. The simulation parameters are listed in Table I.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>SIMULATION PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna configuration</td>
<td>4×1</td>
</tr>
<tr>
<td>Modulation</td>
<td>QPSK</td>
</tr>
<tr>
<td>Channel model</td>
<td>Markov channel model</td>
</tr>
<tr>
<td>Channel offset factor (( \alpha ))</td>
<td>0.99, 0.97, 0.95</td>
</tr>
</tbody>
</table>

Fig. 3 to Fig. 5 compare the performance of three CIOD decoders (SML, JML, and the proposed one) over three different channel conditions such as \( \alpha = 0.99 \), \( \alpha = 0.97 \), and \( \alpha = 0.95 \), where \( \alpha = 0.99 \) and \( \alpha = 0.95 \) are used for modeling slow and fast fading channels, respectively [12].

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>COMPUTATIONAL COMPLEXITY OF VARIOUS DETECTORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detection schemes</td>
<td>Product</td>
</tr>
<tr>
<td>Conventional scheme</td>
<td>128</td>
</tr>
<tr>
<td>Proposed scheme (5dB)</td>
<td>277.16</td>
</tr>
<tr>
<td>Proposed scheme (15dB)</td>
<td>266.24</td>
</tr>
<tr>
<td>Joint-ML</td>
<td>17152</td>
</tr>
</tbody>
</table>

Fig. 3 to Fig. 5 compare the performance of three CIOD decoders (SML, JML, and the proposed one) over three different channel conditions such as \( \alpha = 0.99 \), \( \alpha = 0.97 \), and \( \alpha = 0.95 \), where \( \alpha = 0.99 \) and \( \alpha = 0.95 \) are used for modeling slow and fast fading channels, respectively [12].
In the simulation results, the performance of SML detector shows severe performance degradation, since the decoder suffers from the interference due to time-varying channel. The performance of the proposed algorithm is close to that of JML. In the high mobility environment, shown in Fig. 5, there is a slight difference in performance between JML and the proposed algorithm due to strong interference effect.

We also evaluate the computational complexity among the proposed and conventional algorithms. To estimate the computational efforts, real operations of detection algorithm are counted. Fig. 6 exhibits probability distribution of each computational efforts. In this figure, the complexity of the SML is placed in the lowest position. It is of interest to observe that the computational effort of proposed detection is close to that of the SML. Although the figure is drawn to log-scale, the complexity of the JML shows a distinguished gap from that of other decoders. The figure shows the complexity of the proposed algorithm is not fixed. This is because the proposed scheme is based on iteration and the number of iteration is changed depending on the channel condition. To compare complexity in detail, Table II shows a specific number of operations. The result is measured when SNRs are 5 dB and 15 dB. In the table, the second column represents amount of multiplication between real numbers and the third column shows the number of addition for each detector. The results show that the computational efforts of the proposed algorithm is significantly lower than that of the JML.

V. CONCLUSION

The conventional SML for STBC-CIOD, shows performance degradation in time-varying channel. To resolve the problem, we proposed a novel near-ML decoding algorithm with remarkably reduced complexity. The proposed algorithm considers the interference caused by the loss of quasi-orthogonality and searches near-optimal sequence iteratively. By early termination of the iteration, unessential computational efforts can be avoided. Simulation results demonstrate that the performance of the proposed algorithm is close to that of JML while reducing the computational effort significantly. In conclusion, the proposed algorithm can be a practical alternative to JML for the efficient decoding of STBC-CIOD used for mobile application.

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