LETTER

Low-Complexity Equalizer for OFDM Systems in Doubly-Selective Fading Channels

Namjeong LEE†, Hoojin LEE††, Joonhyuk KANG†††, Members, and Gye-Tae GIL††††, Nonmember

SUMMARY In this letter, we propose a computationally efficient equalization technique that employs block minimum mean squared error (MMSE) depending on $\text{LDL}^H$ factorization. Parallel interference cancellation (PIC) is executed with pre-obtained output to provide more reliable symbol detection. In particular, the band structure of the frequency domain channel matrix is exploited to reduce the implementation complexity. It is shown through computer simulation that the proposed technique requires lower complexity than the conventional algorithm to obtain the same performance, and that it exhibits better performance than the conventional counterpart when the same complexity is assumed.

key words: OFDM, equalization, time varying channel, intercarrier interference

1. Introduction

Orthogonal frequency division multiplexing (OFDM) has attracted a lot of attention as a promising candidate scheme for next-generation wideband wireless systems [1]–[3]. By adding a cyclic prefix (CP) in front of each OFDM symbol block, one-tap equalizer is enough to support OFDM systems in time-invariant multipath channels. When the channel becomes time-varying, however, the one-tap equalizer is not enough because it cannot cancel the intercarrier interference (ICI) caused by the mobility [1]–[4].

Many papers have tried to reduce the effect of ICI [5]–[10]. More specifically, both serial zero-forcing (ZF) and minimum mean squared error (MMSE) equalizers that separately equalize each subcarrier were introduced in [5]–[7]. On the other hand, block ZF and MMSE detectors that jointly equalize all the subcarriers were proposed in [8] and [9]. In [10], the ICI problem has been solved by performing parallel interference cancellation (PIC) at the receiver. However, most of the ICI reduction schemes are highly complex due to the large-sized matrix inversion. Thus, low complexity block MMSE equalizer that exploits band structure of the frequency domain channel matrix and $\text{LDL}^H$ factorization [11] for inversion was introduced in [9]. This equalizer, however, achieves the computational efficiency at the cost of the degradation of the performance.

In this letter, we propose an efficient two-step equalizer that brings performance improvement with low complexity. In the first step, the proposed method applies block MMSE equalizer based on $\text{LDL}^H$ factorization. In the second step, PIC is performed with the pre-obtained output from the first step to provide more reliable detection. Note that the band structure of frequency domain channel matrix is exploited in both procedures of the proposed equalizer, where the bandwidth of the channel matrix is equally or differentively selected in each procedure. We show via simulation results and numerical analysis that the proposed algorithm gives a complexity advantage over the conventional methods under the same performance level, and vice versa.

Notation: A bold face letter denotes a vector or a matrix; $\mathbf{1}$ the transpose of a vector or matrix; $\mathbf{I}$ the conjugate transpose; $[a]_{p,q}$ the element in the $p$th row and $q$th column of $\mathbf{A}$; $\mathbf{I}_{N_A}$ the $N_A \times N_A$ identity matrix.

2. Signal Model

Considering $N$-point FFT/inverse FFT size, the $N \times 1$ received vector $\bar{\mathbf{Y}}$ in frequency domain for the $i$-th OFDM symbol is given by

$$\bar{\mathbf{Y}}[i] = \mathbf{G}[i]\bar{\mathbf{X}}[i] + \bar{\mathbf{Z}}[i]$$  \hspace{1cm} (1)

where $\mathbf{G}[i]$ is the $N \times N$ frequency domain channel matrix, $\bar{\mathbf{X}}[i]$ is the $N \times 1$ transmitted OFDM symbol block, and $\bar{\mathbf{Z}}[i]$ denotes zero-mean complex white Gaussian noise. Assume that only $N_A$ subcarriers are assigned to carry data. Then, the number of unused subcarriers including guard subcarriers equals $N_V = N - N_A$, and hence $\bar{\mathbf{X}}[i]$ in (1) can be written as $\bar{\mathbf{X}}[i] = [\mathbf{0}_{1\times N_V}/2 \; \bar{\mathbf{X}}[i]^T \; \mathbf{0}_{1\times N_V}/2]^T$, where $\bar{\mathbf{X}}[i]^T$ is the $N_A \times 1$ data vector. Here, if we consider the actual data from receive antenna and drop the block index $i$ for notional convenience, (1) reduces to

$$\mathbf{Y} = \mathbf{G} \bar{\mathbf{X}} + \mathbf{Z}$$  \hspace{1cm} (2)

where $\mathbf{Y}$ and $\mathbf{Z}$ are $N_A \times 1$ vectors that are replicas of middle part of $\bar{\mathbf{Y}}[i]$ and $\bar{\mathbf{Z}}[i]$, respectively, and $\mathbf{G}$ is obtained by selecting the $N_A \times N_A$ central block of $\mathbf{G}[i]$. Note that in a time-frequency-selective (i.e., doubly-selective) channel, the matrix $\mathbf{G}$ may be approximated as a band matrix which contains non-zero off-diagonal elements.

Throughout the paper, it is assumed that $E[\mathbf{X}\mathbf{X}^H] = \sigma_z^2 \mathbf{I}_{N_A}$ and $E[\mathbf{Z}\mathbf{Z}^H] = \sigma_z^2 \mathbf{I}_{N_A}$. Then, the signal-to-noise ratio (SNR) is given by $\frac{E[\mathbf{G}^H\mathbf{G}]}{\sigma_z^2}$, where $E[\mathbf{G}^H\mathbf{G}]$ denotes the Frobenius norm of $\mathbf{G}$. 

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3. Low-Complexity Equalization

3.1 Conventional Method

Here, we overview two conventional schemes proposed for ICI cancellation: linear block MMSE equalizer [6] and band-structured block MMSE equalizer [9]. First, the linear block MMSE equalizer is expressed as

\[ \hat{X}_{\text{MMSE}} = G^H(GG^H + \gamma^{-1}I_{N_k})^{-1}Y \]  

(3)

where \( \gamma = \sigma_w^2/\sigma_v^2 \). The right-hand side of (3) requires a non-trivial inversion of the \( N_A \times N_A \) matrix with \( O(N_A^3) \) operations. Second, the band-structured block MMSE equalization in [9] is expressed as

\[ \hat{X}_{\text{LDL}} = D_Q(G)^H K^{-1} Y \]  

(4)

where \( K = D_Q(G)D_Q(G)^H + \gamma^{-1}I_{N_k} \) and \( D_Q(G) \) is a band matrix with the left and right half-bandwidth of \( Q \) (i.e., the bandwidth of the matrix equals \( 2Q + 1 \)). In (4), the parameter \( Q \) determines the performance and complexity of the equalizer; With a large \( Q \) better performance can be achieved, but higher complexity is inevitable at the receiver. Since \( K \) is Hermitian-symmetric, the LDL\(^H\) factorization [11] can be used to implement the inversion \( K^{-1} \). The LDL\(^H\) factorization of the band matrix \( K \) is summarized as follows.

\[ L = I_{N_k}; D = D_Q(K); \nu = 0_{N_A \times 1} \]

for \( j = 1 : N_A \)

\[ m = \max\{1, j - 2Q\}; n = \min\{j + 2Q, N_A\} \]

for \( i = m \cdot j - 1 \)

\[ [v]_i = [L]_{i,i}^H[D]_{i,i} \]

end

\[ [v]_j = [K]_{j,j} - [L]_{j,m-1}^H[v]_{m-1,j}; [D]_{j,j} = [v]_j \]

\[ [L]_{j+1,n,j} = [K]_{j+1,n,j} - [L]_{j+1,n,m-1}^H[v]_{m-1,j} \]

end

Note that, for a small \( Q \), the band-structured block MMSE equalizer may not obtain an acceptable performance, although it is computationally efficient.

3.2 Proposed Equalization Scheme

To overcome the disadvantage of the conventional scheme in [9], we propose an efficient equalization technique that can mitigate the performance loss of the conventional scheme by adopting an additional PIC. In the same manner as the conventional method, the proposed technique employs a block MMSE equalizer that relies on LDL\(^H\) factorization. The additional PIC step is based on the following revised signal model of (2):

\[ Y = D_0(G)X + (G - D_0(G))X + Z. \]  

(5)

The proposed algorithm can be summarized as follows.

<table>
<thead>
<tr>
<th>Step</th>
<th>CM</th>
<th>CA</th>
<th>CD</th>
<th>CS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>((2Q^2 + 3Q + 1)N_A)</td>
<td>((2Q^2 + Q + 1)N_A)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>((2Q^2 + 3Q)N_A)</td>
<td>((2Q^2 + Q)N_A)</td>
<td>(2QN_A)</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>(4QN_A)</td>
<td>(4QN_A)</td>
<td>(N_A)</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>((2Q + 1)N_A)</td>
<td>(2QN_A)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>(2QN_A)</td>
<td>((2Q' - 1)N_A)</td>
<td>(N_A)</td>
<td>(N_A)</td>
</tr>
</tbody>
</table>

Total \((4Q^2 + 12Q + 2Q' + 2)N_A\) \((4Q^2 + 8Q + 2Q')N_A\) \((2Q + 2)N_A\) \(N_A\)
4. Simulation Results

Computer simulations have been run to examine the performance of the proposed technique and the conventional counterpart focusing on the bit error rate (BER). The parameters used in the simulation are summarized in Table 2, and the L-tap exponential delay profile [12] used is represented by

\[ \sigma_p^2 = e^{-p/L} \sum_{q=0}^{L-1} e^{-q/L}. \]

In the simulation, it was assumed that \(|G|^2_F = 1\). The BER values were estimated through 100 channel realizations by transmitting 10,000 OFDM symbols.

Table 3 shows the CM operations required for the conventional method and the proposed technique to obtain the same BER of 0.0015 at SNR = 30 dB. In the table, the relative operation rate means the ratio of the number of CM of the corresponding method to that of the other. From Table 3, it is observed that the proposed algorithm greatly reduces the computational complexity compared with the band-structured block MMSE equalizer. To confirm the validity of the values of \(Q\) and \(Q'\) in Table 1, BER was evaluated with different values of SNR as shown in Fig. 1. From the figure, it is confirmed that the two methods with the respective values of \(Q\) and \(Q'\) have the same BER values for the SNR up to 40 dB.

To confirm the performance advantage of the proposed technique over the conventional method, the BER performance was evaluated assuming that both equalizers have equal complexity. In comparing the complexities of the two algorithms, only the CM operation was considered because the CA and CS requires much less computational resources and the number of CD operations in Table 1 is much smaller than that of CM. Thus, we establish the following equation

\[ (4Q_1^2 + 12Q_1 + 2)N_A = (4Q_3^2 + 12Q_3 + 2)N_A \]

where \(Q_1\) is the half-bandwidth of the channel matrix for the conventional equalizer, and \(Q_3\) and \(Q_3\) are the half-bandwidths of the two channel matrices for the proposed method. Table 4 shows the values of \(Q\) and \(Q'\) chosen to have the same complexity of both methods and the corresponding BER at SNR = 30 dB. Figure 2 shows the simulation results when \((Q_1, Q_2, Q_3)\) is \((3, 2, 16)\). It is observed in the figure that the proposed method has lower BER than the conventional banded Block MMSE equalizer for the SNR range considered in the simulation. The error floors observed in Fig. 2 as well as Fig. 1 are due to the neglected off-diagonal terms of the channel matrix which is in reality non-zero.

The impact of imperfect knowledge of the channel on the performance of the proposed algorithm was examined by using a modified signal model given by \(Y = (G + U)X + Z\), where \(U\) is the channel estimation error matrix whose elements have zero-mean complex Gaussian with variance of \(\sigma_U^2\). The simulation results reveal that the proposed technique has better BER than the conventional scheme as long as \(\sigma_U^2\) is less than 0.1 \(|G|^2_F\).

5. Conclusion

We have proposed a computationally efficient equalizer for OFDM systems. The proposed method employs block MMSE, which relies on \(LDL^H\) factorization. In addition, PIC is executed with pre-obtained output to provide more reliable symbol detection. As a result, compared with the existing block MMSE equalizer, the proposed method can fur-

![Fig. 1 BER comparison between two equalizers at same performance.](image-url)
ther improve the BER performance. The simulation results demonstrated that the proposed algorithm gives advantage in complexity over the conventional method under the same performance level, and vice versa, which obviously validate the effectiveness of the proposed detection algorithm.

Table 4  Performance for total operations (CM)=72 at SNR=30 dB.

<table>
<thead>
<tr>
<th>Method</th>
<th>Q'</th>
<th>Relative operation rate(%)</th>
<th>BER at 30 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDL [^{[9]}]</td>
<td>3</td>
<td>≃100</td>
<td>0.0020</td>
</tr>
<tr>
<td>Proposed Algorithm</td>
<td>2</td>
<td>≃100</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

Fig. 2  BER comparison between two equalizers at same complexity.

References