LETTER

Recursive Decoding for OFDM Systems with Multiple Transmit Antennas

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SUMMARY In this letter, we propose a recursive space time decoding method for orthogonal frequency division multiplexing (OFDM) systems exploiting multiple transmit antenna diversity when the channels are fast fading. We first develop a computationally efficient space-time decoding method involving a matrix inversion to mitigate the channel variation effect. We then further reduce the computational complexity of the matrix inversion decoding method via a recursive formulation. Computer simulation results show that the proposed recursive decoding has much better BER performance than Alamouti decoding, requiring much less computation than the matrix inversion decoding. Moreover, the relative advantage in BER performance of the proposed scheme over Alamouti decoding stands out as the Doppler frequency increases.

key words: equalization, interchannel interference, OFDM, space-time decoding

1. Introduction

Transmit diversity technique has been shown to be effective in combating channel fading, introducing redundancy only in space across multiple antennas, but neither in time nor in frequency [1], [2]. In particular, the Alamouti code is popular and practical thanks to its simple decoding at the receiver [3]. When the Alamouti code is applied to orthogonal frequency division multiplexing (OFDM) systems with multiple transmit antennas, simple decoding at the receiver is based on the assumption that subchannels are flat fading, decoupled, and quasi-static so that each subchannel does not change over a frame and may change from one frame to another. When the channels are fast fading, however, the performance degradation due to interchannel interference (ICI) is severe.

In an effort to mitigate the performance degradation due to the channel time variation, a high complexity time domain block linear filtering method was proposed in [4]. The block linear filtering method requires computing the inverse of a $2N \times 2N$ matrix which amounts to $8N^3$ complex multiplications for each codeword period. The parameter $N$ stands for the FFT size in the system. In [5], we proposed a decision directed space-time decoding method to mitigate the time-varying channel effect. However, the performance improvement was quite limited.

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In this letter, we develop a computationally efficient space-time decoding method involving a matrix inversion of size much smaller than $2N \times 2N$, thereby reducing complexity. Our matrix inversion decoding method is based on the block diagonal approximation in [6]. To further reduce the complexity of the proposed matrix inversion decoding method, we formulate a recursive decoding method. Simulation results demonstrate that the proposed recursive decoding method successfully reduce the computational complexity with negligible performance degradation when compared with matrix inversion decoding.

2. System Description

In this letter, we assume OFDM systems with two transmit antennas and one receive antenna. The bandwidth ($B = \frac{f_T}{2}$) is divided into $N$ equally spaced subcarriers at frequencies $k\Delta f$ with $\Delta f = B/N$ and $k = 0, 1, \ldots, N - 1$. According to the Alamouti code, $X_1[k] X_2[k]$ is transmitted from the two antennas simultaneously during the first symbol period ($l = 1$) for each $k \in K$. During the second symbol period ($l = 2$), $[X_2^*[k] X_1^*[k]]$ is transmitted from the two antennas for each $k \in K$. The set $K \doteq \left\{ \frac{N-N_c}{2}, \ldots, \frac{N+N_c}{2} - 1 \right\}$ is the set of data carrying subcarrier indices; $N_c$ is the number of subcarriers carrying data; $N$ is the FFT size; and consequently the number of virtual carriers is $N-N_c$. We assume half of the virtual carriers are on both ends of the spectrum band. The FFT converts each $N \times 1$ complex vector into a time domain signal and the copy of the last $D$ samples are appended as prefix (cyclic prefix). The time domain transmitted signals from antenna $i$ during the $l$-th symbol period, $x_{il}[n], 0 \leq n \leq N+D-1$, $i \in \{1, 2\}, l \in \{1, 2\}$, are expressed as

$$x_{il}[n] = \sum_{k \in K} S_{il}[k]e^{j2\pi k(n-D)/N},$$

where $S_{il}[k]$ denotes a symbol transmitted from the $i$-th antenna over the $k$-th subchannel during the $l$-th symbol period in an Alamouti codeword. The signals from the two transmit antennas go through different channels. The received signals at the receiver side during an Alamouti codeword period are

$$y_l[n] = \sum_{i=1}^{2} \sum_{p=0}^{L-1} h_{ip}[n, p]x_{il}[n-p] + w_l[n], \quad l \in \{1, 2\}$$

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where \( w_t[n] \) is a circularly symmetric zero-mean white complex Gaussian random process during the \( l \)-th symbol period and \( h_{l,p}[n, p] \) denotes the \( p \)-th multi-path component amplitude from the \( i \)-th transmit antenna at time \( n \) during the \( l \)-th symbol period. The maximum delay spread of the two channels between the two transmit antennas and the receive antenna are assumed to be the same as \( L \). It can be observed that the received signals are the superposition of signals from the two transmit antennas. If a cyclic prefix (CP) of proper length is used, the demodulated signals in the frequency domain via FFT are expressed as follows

\[
Y_l[k] = \sum_{i=1}^{N-1} \sum_{m=0}^{L-1} X_{l,i}[m]H_{l,i,p}[k-m]e^{-j\pi pm/N} + W_l[k]
\]

(3)

where \( l \in \{1, 2\} \) is the OFDM symbol index; \( H_{l,i,p}[k-m] \) represents the FFT of a time-variant \( p \)-th multipath component between the \( i \)-th transmit antenna and the receive antenna during the \( l \)-th symbol period and is defined as

\[
H_{l,i,p}[k-m] \overset{\Delta}{=} \sum_{n=0}^{N-1} h_{l,i,p}[n, p]e^{-j\pi(k-m)n/N}.
\]

(4)

If we define \( a_{ij}[k, m] \overset{\Delta}{=} \sum_{p=0}^{L-1} H_{l,i,p}[k-m]e^{-j2\pi kp/N} \), then \( a_{ij}[k, m], m \neq k \) denotes the ICI from the \( m \)-th subchannel to the \( k \)-th subchannel [6].

3. Space Time Decoding Schemes

In this section, space time decoding schemes are described. First, the Alamouti decoding is briefly reviewed. Second, an extended decoding scheme based on the work [6] is explained. Finally, a recursive decoding scheme is formulated to reduce the complexity of the extended scheme.

3.1 The Alamouti Decoding

If the channel is slow fading, the ICI terms are not significant and

\[
a_{ij}[k, m] \approx 0 \quad \forall m \neq k.
\]

(5)

As a result, the received signal (3) can be split into \( N_c \) small simultaneous equations

\[
Y_{a}[k] = A_{a_{1,2}}[k, k]X_{a}[k] + W_{a}[k], \quad k \in \mathcal{K}
\]

(6)

where

\[
A_{a_{1,2}}[k, k] = \begin{bmatrix} a_{1,2}[k, k] & a_{2,1}[k, k] \\ a_{2,1}[k, k] & -a_{1,2}[k, k] \end{bmatrix}
\]

(7)

\[
Y_{a}[k] = \begin{bmatrix} Y_1[k] \\ Y_2[k] \end{bmatrix}, \quad X_{a}[k] = \begin{bmatrix} X_1[k] \\ X_2[k] \end{bmatrix}^T, \quad \text{and} \quad W_{a}[k] = \begin{bmatrix} W_1[k] \\ W_2[k] \end{bmatrix}^T.
\]

By the assumption that the channels do not change over two symbol periods, \( a_{1,2}[k, k] = a_{1,2}[k, k] = a_{1}[k] \) and \( a_{2,1}[k, k] = a_{2,1}[k, k] = a_{2}[k] \). Space-time decoding is performed simply \( X_{a}[k] = A_{a_{21}}^{-1}[k, k]Y_{a}[k] = \left(\alpha_1[k]^2 + |\alpha_2[k]|^2\right)X_{a}[k] + A_{a_{21}}^{-1}[k, k]W_{a}[k] \). Note that the two symbols in \( \hat{X}_{a}[k] \) are decoupled from each other. The final decisions are made independently

\[
\hat{X}_{a}[k] = \arg \min_{\hat{x}} \|\hat{x} - \rho X_{a}[k]\|, \quad i \in \{1, 2\}
\]

(8)

where \( \rho \approx |\alpha_1[k]|^2 + |\alpha_2[k]|^2 \).

3.2 Decoding via a Matrix Inversion

When the channel is fast fading, however, the approximation (5) no longer holds. It was shown in [6] that when the Doppler frequency and the symbol period product is small, (5) no longer holds. It was shown in [6] that when the

\[
Y_{m1}[k] \approx \begin{bmatrix} \alpha_{m11}[k] & \alpha_{m21}[k] \\ \alpha_{m22}[k] & \alpha_{m12}[k] \end{bmatrix} \begin{bmatrix} X_{m1}[k] \\ X_{m2}[k] \end{bmatrix}
\]

(9)

\[
+ \begin{bmatrix} W_{m1}[k] \\ W_{m2}[k] \end{bmatrix}
\]

where \( Y_{m1}[k] \overset{\Delta}{=} \left[ Y_{1}[k-q], \ldots, Y_{1}[k+q] \right]^T \), \( X_{m1}[k] \overset{\Delta}{=} \left[ X_{1}[k-q], \ldots, X_{1}[k+q] \right]^T \), \( W_{m1}[k] \overset{\Delta}{=} \left[ W_{1}[k-q], \ldots, W_{1}[k+q] \right]^T \) for \( i, l \in \{1, 2\} \) and \( A_{m1l}[k] \) is defined in (10) which is at the top of the next page.

We can estimate the transmitted symbols from (9) via a matrix inversion. If \( 2q \) is the number of subchannels causing ICI, matrix inversion of size \((4q + 2) \times (4q + 2)\) is required for the symbol estimation for each \( k \in \mathcal{K} \).

\[
\hat{X}_{m1}[k] = \begin{bmatrix} \alpha_{m11}[k] & \alpha_{m21}[k] \\ \alpha_{m22}[k] & \alpha_{m12}[k] \end{bmatrix}^{-1} \begin{bmatrix} Y_{m1}[k] \\ Y_{m2}[k] \end{bmatrix}
\]

(11)

The transmitted symbols are estimated by selecting the element at the center of \( \hat{X}_{m1}[k], i \in \{1, 2\} \) and other elements are discarded as in [6]. Note that the entries in \( A_{m1l} \) need to be set as “0” if the column index is among virtual carrier indices. This letter also makes assumption about the parameter \( q \) as \( 2q \leq N - N_c \).

3.3 A Recursive Decoding

In this section, the required matrix inversion size \((4q + 2) \times (4q + 2)\) in 3.2 is lowered to \((2q + 2) \times (2q + 2)\) by a recursive decoding. If we assume we have recovered symbols \( X_i[m], i \in \{1, 2\} \in \mathcal{K}, m < k \) correctly by the time we try to recover \( X[k], i \in \{1, 2\} \), then the effects of these symbols can be subtracted from the received signals as follows

\[
\hat{Y}_{m1}[k] = Y_{m1}[k] - \begin{bmatrix} \alpha_{m11}[k] & \alpha_{m21}[k] \\ \alpha_{m22}[k] & \alpha_{m12}[k] \end{bmatrix} \begin{bmatrix} X_{1}[k] \\ X_{2}[k] \end{bmatrix}
\]

(12)
The following recursive decoding scheme is formulated

$$A_{ml}[k] = \begin{bmatrix} a_{ij}[k-q, k-q] & \cdots & a_{ij}[k-q, k] & 0 & \cdots & 0 \\ a_{ij}[k-q+1, k-q] & \cdots & 0 & a_{ij}[k+q] \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & a_{ij}[k+q, k] & a_{ij}[k+q, k+q] \end{bmatrix}$$

(10)

where $Y_i[l][k] \triangleq \begin{bmatrix} Y[l][k], \cdots, Y[l][k+q-1] \end{bmatrix}^T$, $l \in \{1, 2\}$, $\tilde{Y}_i[l][k] \triangleq \begin{bmatrix} \tilde{Y}[l][k], \cdots, \tilde{Y}[l][k+q-1] \end{bmatrix}^T$, $l \in \{1, 2\}$, $\tilde{X}_i[k] \triangleq \begin{bmatrix} \tilde{X}_i[k-q], \cdots, \tilde{X}_i[k-1] \end{bmatrix}^T$, $l \in \{1, 2\}$, and $A_{ml}[k]$ is defined in (13) which is at the top of the next page. We note that $X_i[k], i = 1, 2$ are recovered in the order of $k = (N-N_c)/2$, $k = (N-N_c)/2+1$, $k = (N-N_c)/2+2$, ..., $k = (N+N_c)/2-1$. Therefore, we have estimated values $\tilde{X}_i[m], m < k$ by the time we try to recover $X_i[k]$.

Once we have subtracted the effects of previous symbols, the following equation is derived.

$$\begin{bmatrix} Y_r[l][k] \\ Y_{r2}[l][k] \end{bmatrix} = \begin{bmatrix} A_{l11}[k] & A_{l21}[k] \\ A_{r22}[k] & -A_{r12}[k] \end{bmatrix} \begin{bmatrix} X_r[l][k] \\ X_{r2}[l][k] \end{bmatrix} + \begin{bmatrix} W_r[l][k] \\ W_{r2}[l][k] \end{bmatrix}$$

(14)

where $Y_i[l][k] \triangleq \begin{bmatrix} Y[l][k], \cdots, Y[l][k+q-1], Y[l][k+q] \end{bmatrix}^T = \begin{bmatrix} Y_i[l][k] \end{bmatrix}^T$, $X_i[k] \triangleq \begin{bmatrix} X[l][k], \cdots, X[l][k+q] \end{bmatrix}^T$, $W_i[l][k] \triangleq \begin{bmatrix} W[l][k], \cdots, W[l][k+q] \end{bmatrix}^T$ for $i, l \in \{1, 2\}$ and $A_{ml}[k]$ is defined in (15) which is at the top of the next page.

As observed in (14), now the required matrix inverse size for symbol estimation is $(2q+2) \times (2q+2)$. By the subtraction of ICI’s from previous symbols (12) and the relation (14), the following recursive decoding scheme is formulated.

For $k = (N-N_c)/2 : (N+N_c)/2-1$

Step 1 : ICI subtraction using (12)

Step 2 : Symbol Decoding

$$\begin{bmatrix} \tilde{X}_r[l][k] \\ \tilde{X}_{r2}[l][k] \end{bmatrix} = \begin{bmatrix} A_{l11}[k] & A_{l21}[k] \\ A_{r22}[k] & -A_{r12}[k] \end{bmatrix}^{-1} \begin{bmatrix} Y_r[l][k] \\ Y_{r2}[l][k] \end{bmatrix}$$

Take $\tilde{X}_i[k], i = 1, 2, k \leftarrow k + 1$.

We note that as the index $k$ is updated after the Step 2, $A_{ml}[k], \tilde{X}_i[k],$ and $Y_i[l][k], i, l = 1, 2$ are updated recursively.

4. Simulation Results

In this section, simulation experiments are conducted for an OFDM system with two transmit antenna and one receive antenna to compare the decoding schemes in Sect. 3. Exact channel estimation at the receiver is assumed, where the bandwidth is $B = 800$ kHz, the FFT size $N = 128$, the CP length $D = 32$, the number of data carrying subchannels $N_c = 120$, consequently the number of virtual carriers is $N - N_c = 8$, and OFDM symbol period $(N + D)T_s = 200 \mu s$. Four subchannels on both ends of the spectrum are not used for data transmission. The subcarriers are modulated by QPSK symbols. The performance criterion is bit error rate (BER) vs. signal to noise ratio (SNR). The total signal power from the two transmit antennas is used for the calculation of SNR. The mobile channel used for simulation is a two path channel with equal power and delays of 0 and 4 $T_s$ respectively, each path experiencing independent Rayleigh fading. Jakes’ model was used for the Ralyleigh fading channel simulation [7].

Fig. 1 BER performance when $f_D = 50$ Hz.

Fig. 2 BER performance when $f_D = 100$ Hz.
\[ A_{i_1}[k] \triangleq \begin{bmatrix} a_{i_1}[k, k-q] & \cdots & \cdots & a_{i_1}[k, k-1] \\ 0 & a_{i_1}[k-1, k-q+1] & \cdots & a_{i_1}[k-1, k-1] \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & a_{i_1}[k + q - 1, k-1] \end{bmatrix} \]  

(13)

\[ A_{i_2}[k] \triangleq \begin{bmatrix} a_{i_2}[k,k] & a_{i_2}[k,k+1] & \cdots & a_{i_2}[k,k+q] \\ a_{i_2}[k+1,k] & a_{i_2}[k+1,k+1] & \cdots & a_{i_2}[k+1,k+q] \\ \vdots & \vdots & \ddots & \vdots \\ a_{i_2}[k+q,k] & \cdots & a_{i_2}[k+q,k+q] \end{bmatrix} \]  

(15)

Fig. 3 BER performance when \( f_D = 200 \) Hz.

Figs. 1–3 compares the three decoding schemes when the Doppler frequency \( (f_D) \) is 50, 100, and 200 Hz. The parameter \( q = 1 \) is used for both the matrix inversion decoding and the recursive decoding. With the parameter \( q = 1 \), the matrix inversion scheme requires a \( 6 \times 6 \) matrix inversion, for each symbol estimation while the recursive decoding scheme requires a \( 4 \times 4 \) matrix inversion. With the less computational complexity, the recursive decoding scheme still has the advantage of better BER performance over the Alamouti decoding, almost comparable to the matrix inversion scheme. It can also be seen from Figs. 1–3 that the BER performance improvement effect of the recursive decoding scheme as well as the matrix inversion scheme tends to stand out as the Doppler frequency becomes larger.

5. Conclusion

In an effort to mitigate the channel variation effect in the OFDM systems with multiple transmit antennas in a computationally efficient manner, we proposed a matrix inversion space-time decoding method. The matrix inversion decoding method is based on block diagonal approximation. We then further reduced the computational complexity of the proposed decoding method via a recursive formulation. The computer simulation results depicted in Figs. 1–3 show that, irrespective of the Doppler frequency, the proposed space-time recursive decoding scheme has the BER performance improvement effect over the Alamouti decoding, which is comparable to that of the matrix inversion scheme, while its computational complexity is much less than that of the matrix inversion scheme.

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References