Abstract—In this paper, a computationally efficient iterative detection algorithm (IDA) for D-STTD systems is presented. To find an optimal sequence by iterative searching, the proposed scheme consists of two stages. Firstly, the IDA establishes the candidate sequences. Then, it iteratively finds an optimal sequence by examining the candidate sequences until the early termination condition is satisfied. The adaptation of the number of candidate sequences makes the proposed IDA more efficient in computational complexity. The proposed detector is shown to achieve the near-ML performance with significantly reduced complexity.

I. INTRODUCTION

It is generally known that multiple antenna systems provide outstandingly high spectral efficiencies in rich scattering environments compared to single antenna systems [1]. By using multiple antennas on both sides of a communication link, diversity gain and/or spatial multiplexing gain can be achieved. Spatial multiplexing communication systems send independent data streams simultaneously to increase data rate (e.g., V-BLAST (Vertical Bell Labs Layered Space-Time) architecture [2]), whereas space-time block codes (STBC) [3] (otherwise called space-time transmit diversity (STTD) in [4]) focuses on the diversity advantage of multiple antennas to improve multiple-input multiple-output (MIMO) system performance. As a hybrid system to exploit both diversity gain and spatial multiplexing gain at the same time, double space-time transmit diversity (D-STTD) systems were introduced (e.g., [5], [6]). It combines two STTD units at the transmitter and employs interference cancellation based detector at the receiver.

In [7], combined interference cancellation and maximum-likelihood (ICML) decoding scheme was suggested in multiuser environment. Since the D-STTD systems are viewed as a special case of multiuser scenario, the ICML detection approach can be directly applied to detect the signals transmitted by D-STTD systems. That is, the ICML first suppresses the inter-unit interference, and then it jointly solves two single detection problems. Although this algorithm shows a reasonable bit error rate (BER) performance at a feasible computational efforts, there is still a considerable performance gap between the ICML and the optimal detection (ML). The ML is known to guarantee the best performance than any other detection schemes at the cost of high complexity. To resolve this complexity problem, various low complexity near- or exact-ML detection schemes have been investigated. As an illustrative example, sphere detection (SD) and its variant were appeared in [8]- [10]. Although the SD provides an optimal performance with tractable complexity, further reduction of the detection complexity is still crucial issue considering size and power limitation of the mobile station.

In D-STTD systems, considering the particular structure of the upper triangular matrix after QR decomposition of the channel matrix, there are no interference between two signals from the same unit but inter-unit interference between two units exists. Not only to maximally exploit the special nature of the D-STTD systems but also to effectively avoid the inter-unit interference, this paper aims to propose a simple iterative detection algorithm (IDA) for D-STTD systems. Firstly, $K$ partial candidate sequences for signals from the second unit are established. After that, starting from the first sequence of the candidate list, the algorithm iteratively finds an optimal sequence until the early termination condition is satisfied. The further complexity saving method that adaptively limits the size of the candidate list is also presented in this paper. As a result, the proposed algorithm provides near-optimal performance with significantly reduced complexity compared with that of SD.

This paper is organized as follows. In Section II, the system model is described. Proposed computationally efficient algorithm is introduced and the further advanced complexity reduction method is investigated in Section III-A and III-B, respectively. The simulation results and computational efforts are given in Section IV and finally concluding remarks are provided in Section V.

Notation: A bold face letter denotes a vector or a matrix; $[.]^*$ denotes conjugate; $[.]^H$ denotes conjugate transpose.

II. SYSTEM DESCRIPTION

Figure 1 describes D-STTD system with four transmit ($n_t = 4$) and two receive ($n_r = 2$) antennas. The data stream $\{b(n)\}$ is demultiplexed into four data substreams. These substreams are mapped into $M$-PSK or $M$-QAM symbols $x(n)$ and divided into groups of two symbols each (i.e., $x_1$, $x_2$ and $x_3$, $x_4$). Then, space-time block coded symbols are transmitted over four antennas during two consecutive symbol periods. At the receiver, signals are received over two consecutive symbol
periods. Then, the system equation can be expressed as

$$\begin{bmatrix} y_{11} \\ y_{12} \\ y_{21} \\ y_{22} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{22} & h_{21} & h_{24} & h_{23} \\ h_{22} & h_{21} & h_{24} & h_{23} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} w_{11} \\ w_{12} \\ w_{21} \\ w_{22} \end{bmatrix}$$

where $y_{ij}$ and $w_{ij}$ are the received signal and noise component at $i$-th receive antenna during $t$-th symbol period, respectively. The vector $w$ denotes zero-mean complex additive white Gaussian noise while the average transmit power of each antenna is normalized to one. The channel matrix $H$ contains uncorrelated complex Gaussian fading gains between the $i$-th receive and $j$-th transmit antennas $h_{ij}$, where $i = 1, 2$ and $j = 1, 2, 3, 4$. The channel information is assumed to be perfectly known to the receiver.

Performing QR decomposition of the channel matrix $H$, we obtain

$$H = QR,$$

where $Q$ is the $4 \times 4$ unitary matrix and $R$ is the $4 \times 4$ upper triangular matrix. By multiplying $Q^H$ to the both sides, (1) is rewritten as

$$\tilde{y} = Rx + \tilde{w}.$$  

Due to the property of the unitary matrix, the statistical properties of $\tilde{w}$ remain unchanged. Considering the unique property of D-STTD system as shown in [11], the upper triangular matrix $R$ in (3) has a special structure as

$$R = \begin{bmatrix} r_{1,1} & 0 & r_{1,3} & r_{1,4} \\ 0 & r_{2,2} & r_{2,3} & r_{2,4} \\ 0 & 0 & r_{3,3} & 0 \\ 0 & 0 & 0 & r_{4,4} \end{bmatrix}.$$  

From the observation that two terms of upper triangular part are zero, we easily see that $x_3$ and $x_4$ transmitted from the second unit do not interfere with each other when two signals are detected. Fortunately, this is the same for $x_2$ and $x_1$ from the first unit. However, there exists inter-unit interference in D-STTD systems which results in error propagation. Not only to maximally exploit the special nature of the D-STTD systems but also to effectively avoid the inter-unit interference, we introduce simple iterative detection algorithm (IDA) for D-STTD systems in the following section.

III. SIMPLE ITERATIVE D-STTD DETECTION

In this section, we provide the details of the proposed IDA. As mentioned earlier, the D-STTD systems have a simple structure inherited from the unit by unit signal processing. Thus, the transmitted signals can be manipulated in two pairs for detection at the receiver. The IDA first establishes $K$ partial candidate sequences for $x_4$ and $x_3$. Note that the signal detection order is reverse to the transmit antenna index due to the property of $R$ matrix. The partial sequences are determined by the combination of the most likely signals corresponding to $x_4$ and $x_3$ each (the details will be described later). After that, the signals from the first unit, $x_2$ and $x_1$, are chosen depending upon the partial candidate sequences selected at the first step. The algorithm iteratively finds an optimal sequence until the early termination condition is satisfied. The proposed IDA has the performance-complexity tradeoff depending upon how many number of the partial candidate sequences are used in detection process. The method illustrating this tradeoff is discussed in subsection B.

A. Algorithm Description

The signal detection in uncoded MIMO systems is usually based on the squared Euclidean distance as a metric. Since there is no exception in our case, we first consider the metric calculation required to detect the D-STTD signals. Thus, the ML criterion from (3) is expressed as

$$\min_{x \in S} \|\tilde{y} - Rx\|^2,$$  

where $S$ is the constellation set. Based on this criterion, we proceed to introduce the step by step procedure.

**Step 1 (candidate list generation):** In this step, $K$ partial candidate sequences are generated by the combination of the most likely signals for $x_4$ and $x_3$ each. The minimum vector norm given in (5) is equivalent to the sum of the squared Euclidean distance of each transmitted signal, $x_1, \ldots, x_4$. For two signals belonging to the second unit, the metric of the $i$-th
constellation point, i.e., $\hat{x}_i^k \in \mathcal{S}$, is calculated as
\[ m_k^i = |y_k - r_{k,k} \hat{x}_i^k|^2, \quad k = 4, 3, \quad i = 1, \cdots, M. \] (6)

By the combinations of the metrics computed by (5), the $K$ ordered list of the partial candidate sequences is established, where the first component of the list has the accumulated smallest metric. Let the ordered list denote $X = [\bar{x}^1, \cdots, \bar{x}^i, \cdots, \bar{x}^{K}]^T$ and the corresponding cumulative metric represent $\mathbf{m} = [\bar{m}^1, \cdots, \bar{m}^i, \cdots, \bar{m}^K]^T$, $\bar{x}^i = [\bar{x}_4^i, \bar{x}_2^i]$ and $\bar{m}^i = m_{4}^i + m_{3}^i$, where $p, q \in \{1, \cdots, M\}$. $\bar{x}^i$ has the smallest metric $\bar{m}^i$ and the maximum value of the sequence size $K$ is $M^2$. We also set variable $T$ as infinity that is used as a threshold in detection process.

Step 2 (detection of the remaining part of signals) : Starting with the first sequence of $X$, $\bar{x}^1$, the remaining signals ($x_2$ and $x_1$) succeeding each candidate sequence are iteratively searched until an optimal full sequence is selected. When the candidate sequence is assumed to be $\bar{x}^l$, the metric for two signals from the first unit is also computed like (6) as
\[ m_{k'}^l = |y_{k'} - R_{k',3:4} (\bar{x}^l)^T - r_{k',k'} \hat{x}_k^{k'}|^2, \quad k' = 2, 1. \] (7)

Then, the optimal full sequence with $\bar{x}^l$ can be represented as $X_{\text{full}} = [\hat{x}_3^l, \hat{x}_2^l, \hat{x}_1^l]$, where $r, s \in \{1, \cdots, M\}$, $\hat{x}_2^r = \min_{\bar{x}_s^r \in S} m_{2}^r$, and $\hat{x}_1^s = \min_{\bar{x}_s^r \in S} m_{1}^s$. The corresponding cumulative metric of $X_{\text{full}}$ is calculated as $m_{\text{full}} = m_{l}^1 + m_{s}^2 + m_{r}^1$. When the current $m_{\text{full}}$ is lower than the threshold $T$, replace $T$ by $m_{\text{full}}$ and take $X_{\text{full}}$ as an optimal sequence. Otherwise, algorithm directly goes to next step without replacements. Larger $m_{\text{full}}$ than $T$ means that the previously detected full sequence is more likely to be an optimal sequence.

Step 3 (termination): By comparing $T$ with $\bar{m}_{l+1}$, the algorithm determines to find more likely sequence than current one with $\bar{x}^{l+1}$ or terminate. When the $\bar{m}_{l+1}$ is smaller than $T$, the current $X_{\text{full}}$ is not an optimal sequence and the algorithm implements Step 2 again with $\bar{x}^{l+1}$ to find more likely sequence. Otherwise, when $\bar{m}_{l+1}$ is larger than $T$, the current $X_{\text{full}}$ is considered as an optimal sequence since $T$ is absolutely smaller than the metrics of all the remaining candidate sequences, $\bar{m}_{l+1}, \cdots, \bar{m}^K$. By setting this early termination condition, the algorithm efficiently finds the optimal full sequence without unnecessary effort of searching all the candidate sequences. The algorithm also terminates after examining all the $K$ candidate sequences.

The flowchart of the proposed algorithm with further complexity saving method introduced in the next subsection is described in Figure 2.

B. Complexity saving methods

One of the advantage of the proposed IDA is the selective number of the candidate sequence, $K$. It provides a reasonable performance-complexity tradeoff. For example, to get an advantage in computational complexity, it is desirable to assign small number for candidate size. In this case, only a small part of the possible sequences being searched, consequently, results in poor performance. On the other hand, accuracy of the detection can be enhanced with large size of sequences with high computational complexity. Although too small $K$ leads to a performance degradation, it is observed that about 49 combinations (i.e. combine 7 most likely constellation points for $x_4$ and $x_3$ each) in searching is enough to achieve a near-optimal performance when the modulation is 16QAM. For further complexity saving, the following two criteria are considered to adaptively select the candidate size $K$.

Criterion 1: The IDA may devote a large amount of effort to find the best sequence at low SNR regime, since there are no radically different values in sequence metrics. The excessive iterative detection leads to longer detection delay and hence the computational complexity becomes high at low SNR regime. However, the situation is reversed at high SNR regime. The received sequence is not too noisy and the metrics between different levels are clearly different. Instead of examining all the pair of sequences, detector almost directly finds the most likely paths. Accordingly, the average computational complexity curve of the proposed IDA exhibits extreme unbalance tendency. Considering that receiver should be designed to cover all possible worst cases, it is undesirable to implement the hardware structure of the system. Hence,
assigning different $K$ according to the SNR will be very helpful for saving the computational complexity at low SNRs.

Criterion 2: When the channel condition is good, due to the minor effect from the fading, the detection will be accurate. In this case, it is enough to use a few memory for searching. Conversely, when the channel condition is bad, the large number of candidate sequences will be needed for searching to increase the detection accuracy. In this manner, assigning different $K$ according with the channel condition will be helpful for reducing the complexity of the algorithm without performance loss. To test the channel condition, we can check the diagonal components of $R$ matrix. Among four diagonal components of $R$ matrix, $r_{4,4}$ (or $r_{3,3}$) dominantly determines the detection accuracy, since the error propagation from the second unit to the first unit can be occurred, considering the structure of $R$.

Summarizing and considering above criteria, as a preprocessing, we propose the $K$ to be adaptively selected according to the SNR and channel condition after QR decomposition. The empirical value of $K$ is summarized in Table I for a specific case to achieve a near-optimal performance with reduced computational complexity.

It is clearly expected that we can find the similar aspect in suitable conditional value of memory size for higher order modulations, such as 64-QAM and 256-QAM. Considering that the computational efforts in detection get higher as the modulation order increases, the proposed algorithm amplifies its benefits in higher order modulation systems.

### IV. Simulation Results

In this section, we compare the performance of the proposed D-STTD detector with other detection schemes in terms of the average bit error rate (BER) performance and the corresponding computational complexity. Since it is shown in [9] that the performance of sphere decoder (SD) [8] is the same as that of ML, we refer the BER performance curve of SD as the performance of the optimal detector. The simulation parameters are listed in Table II.

#### A. Performance Analysis

Figure 3 presents the BER performance of the IDA with various values of $K$ and conditional size assignment. As shown in Figure 3, the BER performance of the IDA approaches that of SD, as the candidate sequence size increases. Moreover, the proposed method utilizing $7^2$ size candidate sequences is enough for searching to achieve near-SD performance. In the case of using adaptive $K$ according to the SNR and channel condition, our proposed algorithm also achieves almost the same performance as $K = 7^2$, even much smaller number of sequences are used in detection processes. Furthermore, the IDA outperforms ICML even with small number of partial candidate sequences.

#### B. Complexity Analysis

To evaluate the computational efforts of the proposed algorithm and compare it with those of SD and ICML, we count the real operations such as multiplication, addition, and division that can be occurred in all the steps to detect the transmitted signals at the receiver. For this calculation, each complex operation is converted into real operation to get a clear idea of complexity. For instance, one complex multiplication is equivalent to three real multiplications and five real additions.

Figure 4 shows the computational complexity of the proposed algorithm and other detection schemes. Considering the BER performance of the proposed algorithm depicted in Figure 3, the proposed IDA provides good tradeoff between performance and complexity. The IDA with $K = 7^2$ and adaptive value of $K$ shows advantage in complexity over SD. Especially, the complexity of IDA with adaptive value of $K$

### TABLE I

<table>
<thead>
<tr>
<th>SNR</th>
<th>$(4,4)$ of $R$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sim 10$dB</td>
<td>$\sim 0.6$</td>
<td>$4^2$</td>
</tr>
<tr>
<td></td>
<td>$0.6 \sim 1$</td>
<td>$2^2$</td>
</tr>
<tr>
<td>$10 \sim 16$dB</td>
<td>$\sim 0.6$</td>
<td>$6^2$</td>
</tr>
<tr>
<td></td>
<td>$0.6 \sim 1$</td>
<td>$4^2$</td>
</tr>
<tr>
<td>$16$dB~</td>
<td></td>
<td>$7^2$</td>
</tr>
</tbody>
</table>
is dramatically lower than that of SD with high favorable performance. The proposed algorithm also shows lower computational efforts than ICML.

V. CONCLUSION

We have proposed a computationally efficient sub-optimal algorithm well-suited to D-STTD systems. The algorithm deals with transmitted signals in two pairs, signals from the first and second unit, and finds the optimal sequence until it satisfies the early termination condition. Moreover, by adaptively choosing memory size in accordance with the channel condition and SNR, the computational complexity of the algorithm is dramatically reduced, especially at low SNRs. The simulation results prove that the proposed algorithm achieves near-optimal performance with a fraction of an effort compared with other detection schemes. We expect that the proposed algorithm can be a practical detection scheme for the D-STTD systems.

REFERENCES