Abstract—In this paper, we study an energy efficient two-way amplify-and-forward (AF) relay network equipped with multiple antennas at both sources and relay. To minimize the transmit power while satisfying quality of service (QoS) of both sources, transmit beamformers and receive combiners are designed with ZF-based relay processing matrix. Due to the non-convexity of the optimization problem, the original problem is decomposed into two simpler optimization problems which can be solved through iteration algorithm. As an alternative, eigen-beamforming based technique is also provided to reduce the complexity of the iterative solution at the expense of the power usage. Numerical results show that the proposed solution efficiently reduces the additional power usage of the system compared to the eigen-beamforming based technique while meeting the QoS constraint of each source. Moreover, we can see the proposed algorithm converges to the steady-state value in a reasonable number of iterations.

I. INTRODUCTION

Recently, relay-based communication has been intensively researched in many literatures to increase the capacity and extend the cell coverage [1]. As a specific model of relay networks, two-way relay channel where two nodes communicate via a relay in two time slots was investigated [2]. For this network, amplify-and-forward (AF) method has been used as a transmission strategy at the relay in many researches [3]–[7] due to its simplicity.

For the two-way AF relay system, two optimization problems can be considered: the sum-rate maximization of the system subject to a power constraint; and the power minimization problem, which is considered in this paper, while satisfying quality-of-service (QoS) of the transceivers. In [3]–[5], sum-rate maximized AF two-way relay systems were studied in various antenna configurations. The optimal beamforming that achieves Pareto-optimal points and achievable sum-rate maximizing AF relay beamforming scheme at high SNR assumption were described in [3] and [4], respectively, when only the relay has multiple-antennas. Also, in [5], the joint design of transceiver and relay which are equipped with multiple-antennas was studied. For the second optimization problem, in [6], the total transmit power was minimized for a distributed two-way relay system having single antenna at both transceivers and distributed relays.

From the review above, to the best of our knowledge, researches of power minimization technique for the two-way AF relay system have not yet actively investigated compared to the sum-rate maximization researches. In this paper, an energy efficient two-way AF relay protocol that minimizes the total transmit power subject to meeting a QoS of each transceiver is studied. We focus on the case where the transceivers and the relay have multiple-antennas while each source transmits a single stream through beamforming. Our approach could be differenciated from the method in [6] in two respects: the first is a design of transmit beamformers and receive combiners for the multiple-antenna sources; and the second is a design of concentrated (i.e., not distributed) relaying matrix. The main contributions of this paper are as follows: (i) Total power minimization technique: We design the transmit beamforming vectors, receive combing vectors and transmit power of each transceiver with zero-forcing (ZF)-based relaying matrix [7]. Since the optimization problem for the given ZF-based relaying matrix is non-convex, we propose to approximate its solution by iterating two simpler optimization problems; (ii) Alternative beamforming strategy: We introduce an eigen-beamforming based solution as an alternative scheme which is simple to implement.

This paper is organized as follows. In Section II, the two-way AF relay network signal model equipped with multiple antennas at the transceivers and the relay is described. Section III presents the total power minimization methods. The Section IV provides numerical results of the total power minimization and convergence performance of the proposed techniques. Finally, Section V presents our conclusions. The explanation of ZF processing matrix is relegated to Appendix.

Notation: A bold face letter denotes a vector or a matrix; [:]T, [:]H, [:]† are the transpose, the conjugate transpose, the pseudo-inverse, and the trace of a matrix, respectively; \(\|A\|_F\) is the Frobenius norm of a matrix \(A\); \(\|a\|\) is the 2-norm of a vector \(a\); \(E[\cdot]\) is an expected value of a vector or a matrix; \(I_q\) is a \(q\)-by-\(q\) identity matrix.

II. SYSTEM MODEL

We consider a wireless network consisting of two transceivers with \(N_t\) antennas, \(S1\) and \(S2\), and a relay node \(R\) having \(N_r\) antennas, as shown in Fig. 1. We assume that due to large path loss effect, there is no direct link between the transceivers. For this reason, two transceivers communicate with each other through the relay node. We herein use two-way AF transmission protocol that uses two-consecutive time-slots. During the first time-slot, the two transceivers transmit message symbols to the relay simultaneously, while in the second time-slot, the relay amplifies its received signals and broadcasts to the two transceivers. We assume that channel

Energy Efficient Two-way AF Relay System with Multiple-antennas

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reciprocity during the first and second time-slots and the channel matrices $H_k$, where $k = 1, 2$, are changed slowly so that keep constant over multiple times of a transmission time and perfectly known at the relay. The received signal at the relay in the first time-slot, $r \in \mathbb{C}^{N_r \times 1}$, can be written as

$$r = \sqrt{P_1}H_1f_1s_1 + \sqrt{P_2}H_2f_2s_2 + n,$$

where $P_1$ and $P_2$ are the transmit power of $S_1$ and $S_2$, respectively; $s_1$ and $s_2$ are the information symbols transmitted by $S_1$ and $S_2$, respectively, with power constraint $\mathbb{E}\{||s_1||^2\} = \mathbb{E}\{||s_2||^2\} = 1$; $f_1, f_2 \in \mathbb{C}^{N_t \times 1}$ are the transmit beamforming vectors at $S_1$ and $S_2$ with power constraint $||f_1||^2 = ||f_2||^2 = 1$; and $H_1, H_2 \in \mathbb{C}^{N_r \times N_t}$ are the channel matrices from $S_1$ and $S_2$ to the relay; and $n \in \mathbb{C}^{N_r \times 1}$ is the noise at the relay with $\mathbb{E}\{nn^H\} = \sigma^2_n I_{N_r}$.

In our system model, we emphasize that each transceiver sends a single data stream and the relay applies AF operation by multiplying linear precoding matrix, $W$, to $r$. Hence, the relay transmit signal $x \in \mathbb{C}^{N_r \times 1}$ can be expressed as

$$x = Wr.$$  

With an assumption of the channel reciprocity, the received signals at $S_1$ and $S_2$ are expressed as

$$y_1 = d_1^H H_1^H x + d_1^H z_1 = \sqrt{P_1}d_1^H H_1^H W H_1 f_1 s_1 + \sqrt{P_2}d_1^H H_1^H W H_2 f_2 s_2 + d_1^H H_1^H W n + d_1^H z_1,$$  

and

$$y_2 = d_2^H H_2^H x + d_2^H z_2 = \sqrt{P_1}d_2^H H_2^H W H_1 f_1 s_1 + \sqrt{P_2}d_2^H H_2^H W H_2 f_2 s_2 + d_2^H H_2^H W n + d_2^H z_2,$$  

where $d_1, d_2 \in \mathbb{C}^{N_t \times 1}$ are the receive combining vectors at $S_1$ and $S_2$, respectively, and we assume $||d_1||^2 = ||d_2||^2 = 1$.

Hence, the objective function also minimized, thereby this condition is

$$\min_{P_1, P_2, f_1, f_2, d_1, d_2, W} P_T \quad \text{s.t.} \quad \text{SNR}_1 \geq \gamma_1, \text{SNR}_2 \geq \gamma_2,$$  

where $\text{SNR}_1$ and $\text{SNR}_2$ are received SNRs at $S_1$ and $S_2$, respectively. The total transmit power $P_T$ can be written as

$$P_T = P_1 + P_2 + P_r,$$  

where $P_r$ is the relay transmit power. Note that

$$P_r = \text{tr}\{\mathbb{E}(xx^H)\} = \text{tr}\{P_1 W H_1 f_1^t H_1^H W^H + P_2 W H_2 f_2^t H_2^H W^H + \sigma^2_n WW^H\}.$$  

In this paper, we apply a linear processing matrix based on ZF criteria in [7] instead of attempting to find the optimal $W$ since finding jointly optimized $W$ is very complicated although it is expected that further power minimization is available by jointly optimizing $W$. In Appendix A, a method to find ZF-based $W$ for given transmit beamformers and receive combiners is described. With ZF-based $W$, self-interference terms, $\sqrt{P_1}d_1^t H_1^H W H_1 f_1 s_1$ and $\sqrt{P_2}d_2^t H_2^H W H_2 f_2 s_2$ in (3) and (4), respectively, are effectively removed at the relay, thus (3) and (4) can be rewritten as

$$y_1 \Delta = \sqrt{P_2}d_1^t H_1^H W H_2 f_2 s_2 + d_1^H H_1^H W n + d_1^H z_1,$$  

and

$$y_2 \Delta = \sqrt{P_1}d_2^t H_2^H W H_1 f_1 s_1 + d_2^H H_2^H W n + d_2^H z_2.$$  

From (8) and (9), we can express the received SNRs of each transceiver in (5) as

$$\text{SNR}_1 = \frac{P_2 d_1^t H_1^t W H_2 f_2^t H_2^t H_1^t W H_1 d_1}{\sigma^2_n d_1^t H_1^t W W^H H_1 d_1 + \sigma^2_{S_1}},$$  

and

$$\text{SNR}_2 = \frac{P_1 d_2^t H_2^t W H_1 f_1^t H_1^t W H_2 d_2}{\sigma^2_n d_2^t H_2^t W W^H H_2 d_2 + \sigma^2_{S_2}}.$$  

Using (6)-(11) and ZF-based $W$, the optimization problem (5) can be rewritten as (12), shown at the bottom of the next page. From the relation of the inequality constraints and the objective function in (12), we can find that the inequality constrains in (12) should be restricted to the equality constraints at the optimal solution. Otherwise, for instance, if the left sides of the inequality constraints are over $\gamma_1$ and $\gamma_2$ at optimal solution, we can reduce $P_1$ and $P_2$ to meet this constraint with equality. However, since this reduced $P_1$ and $P_2$ make objective function also minimized, thereby this condition is
contradictory to the optimality. With the equality constraints, (12) can be rewritten as (13) by using (14) and (15)

\[ P_1 = \gamma_2 \frac{\sigma_d^2 d_1^H H_2^H W W^H H_2 d_2 + \sigma_d^2}{d_2^H H_2^H W H_1 f_1^H H_1^H W^H H_2 d_2} \]  

(14)

and

\[ P_2 = \gamma_1 \frac{\sigma_d^2 d_1^H H_1^H W W^H H_1 d_1 + \sigma_d^2}{d_1^H H_1^H W H_2 f_2^H H_2^H W^H H_1 d_1} \]  

(15)

Since the optimization problem (13) is non-convex, it is difficult to jointly design \( f_1, f_2, d_1, \) and \( d_2 \). For this reason, we divide the original problem into two simpler optimization problems which are convex. Then those problems are iteratively solved. As an alternative scheme, eigen beamforming is also introduced which provides less design complexity than the proposed iterative scheme at the expense of the additional power usage to meet the QoS requirements.

### A. Power Minimization Technique

We propose a power minimization technique to find the suboptimal solution. As aforementioned, due to the non-convexity of the original problem, we attempt to decompose the original problem into two minimization problems as follows:

\[
\begin{align*}
\min_{P_T} \quad & \text{for given } f_1, f_2 \text{ and } W, \text{ s.t. } ||d_k||^2 = 1. \\
\min_{P_T} \quad & \text{for given } d_1, d_2 \text{ and } W, \text{ s.t. } ||f_k||^2 = 1. \\
\end{align*}
\]

(16)

Each optimization problem in (16) is convex, hence easy to solve. Through the iteration of the two problems, we can find the suboptimal solution to minimize the total power.

The optimal \( d_1 \) and \( d_2 \) in the first problem of (16) are trivial solution well-known to be the MMSE filters

\[ \hat{d}_1 = \alpha_1 (H_2^H W W^H H_1 + I_{N_r})^{-1} H_1^H W H_2 f_2 \]

and

\[ \hat{d}_2 = \alpha_2 (H_2^H W W^H H_2 + I_{N_r})^{-1} H_1^H W H_1 f_1, \]

where \( \alpha_1 \) and \( \alpha_2 \) are normalization factors to make \( ||\hat{d}_1||^2 = 1 \) and \( ||\hat{d}_2||^2 = 1 \), respectively. From (13), the second problem in (16) can be rewritten as

\[
\begin{align*}
\min_{f_1, f_2} \quad & \text{tr}\left\{ \frac{n}{a^H f_1 f_2^H a} \left( \frac{1}{N_r} I_{N_r} + C f_1 f_1^H C^H \right) \right\} \\
& + \text{tr}\left\{ \frac{n}{a^H f_2 f_2^H a} \left( \frac{1}{N_r} I_{N_r} + D f_2 f_2^H D^H \right) \right\} \\
& + \text{tr}\left\{ \sigma_d^2 W W^H \right\},
\end{align*}
\]

(17)

where \( a = H_1^H W H_2 d_2, b = H_2^H W H_1 d_1, C = W H_1, D = W H_2, m = \gamma_2 (\sigma_d^2 d_1^H H_1^H W W^H H_2 d_2 + \sigma_d^2), \) and \( n = \gamma_1 (\sigma_d^2 d_1^H H_2^H W W^H H_1 d_1 + \sigma_d^2) \). In (17), The optimal solutions for \( f_1 \) and \( f_2 \) can be obtained by solving the following problems:

\[
\begin{align*}
\min_{f_1} \quad & \text{tr}\left\{ \frac{m}{a^H f_1 f_1^H a} \left( \frac{1}{N_r} I_{N_r} + C f_1 f_1^H C^H \right) \right\} \\
& + \text{tr}\left\{ \frac{m}{a^H f_2 f_2^H a} \left( \frac{1}{N_r} I_{N_r} + D f_2 f_2^H D^H \right) \right\} \\
& + \text{tr}\left\{ \sigma_d^2 W W^H \right\},
\end{align*}
\]

(18)

and

\[
\begin{align*}
\min_{f_2} \quad & \text{tr}\left\{ \frac{n}{b^H f_2 f_2^H b} \left( \frac{1}{N_r} I_{N_r} + D f_2 f_2^H D^H \right) \right\} \\
& + \text{tr}\left\{ \frac{n}{b^H f_1 f_1^H b} \left( \frac{1}{N_r} I_{N_r} + C f_1 f_1^H C^H \right) \right\} \\
& + \text{tr}\left\{ \sigma_d^2 W W^H \right\},
\end{align*}
\]

(19)

The minimization problem in (18) and (19) are known as Rayleigh quotient minimization problem [8]. Therefore, the optimal \( f_1 \) and \( f_2 \) are the eigenvectors which are corresponding to the minimum eigen values of following two generalized eigen value problems \( (I_{N_r} + C^H C) f_1 = \lambda_1 a a^H f_1, (I_{N_r} + D^H D) f_2 = \lambda_2 b b^H f_2 \), respectively, where \( \lambda_1 \) and \( \lambda_2 \) are Lagrange coefficients.

With the optimized solutions above, \( P_1 \) and \( P_2 \) in (14) and (15), respectively, are updated and then \( P_r \) in (7) is calculated with \( P_1 \) and \( P_2 \) as well. The iterative algorithm works as described in Algorithm 1.

### B. Eigenbeamforming

The power minimization technique proposed in Section III-A optimizes the transmit beamformers and the receive combiners at the relay thanks to the perfect channel state information (CSI). Then, the optimized vectors are informed of the sources through the feedback channel. If the CSI between the sources and the relay is available at each source, an alternative technique can be investigated where each source designs own transmit/receive vectors. This scenario could be realized with the channel estimation in downlink channel (i.e., the channel from the relay to the sources).

As a simple power minimization technique, we propose an eigen-beamforming based solution where each source designs
own transmit beamformers as the right singular vector corresponding to the largest singular value of the channel matrix. Considering the reciprocity of the channel matrices, the receive decoder, $d_k$, is the same with the transmit beamformer, $f_k$, $k = 1, 2$.

In one-way AF relay system, it was proved that the eigen-beamforming technique at the source and the destination is the achievable rate maximizing scheme with an optimized relay processing matrix [9]. Maximization of the achievable rate is equivalent to the maximization of the received SNR in one-way AF relay system. Obviously, for the one-way relay system, SNR maximizing technique could be considered as a power-saving technique also since the required QoS is achieved with small power consumption. In this sense, we have a conjecture that eigen-beamforming based technique works as an energy-efficient transmit/receive strategy for the two-way relay system, although we aim to minimize the total transmit power not the individual power of each source.

Since the eigen-beamforming scheme requires relatively low computational complexity for designing transmit beamformers and receive combiners, it can be good alternative power-efficient scheme of the iterative optimization technique proposed in Section III-A at the expense of additional power usage.

IV. NUMERICAL RESULTS

In this section, we present simulation results of the total power minimization technique proposed in this paper. For each frame of the transceivers, a flat fading MIMO channel matrix $H_1$ and $H_2$ are generated from the independent Gaussian random variables with zero mean and unit variance. Channel $H_1$ and $H_2$ are fixed during the one round of the information exchange between the sources. The noise power $\sigma^2_{R}$, $\sigma^2_{S_1}$, and $\sigma^2_{S_2}$ are assumed to be 0 dBW.

![Algorithm 1](image)

Fig. 2. shows the average total transmit power of the proposed power minimization technique, eigen-beamforming and equal-gain beamforming method (i.e., $f_k = d_k = (1/\sqrt{N})[1 \ldots 1]^T$) satisfying QoS of each transceivers, when $N_t=2$ and $N_r=4$. In Fig. 2, the horizontal axis is the required minimum SNR at each transceiver, when $\gamma_1=\gamma_2=\gamma$. From the Fig. 2., we can find that our proposed iteration scheme shows better power minimization performance than equal-gain beamforming and eigen-beamforming method. In low $\gamma$, we can see that the performance of eigen-beamforming scheme is very close to the iteration beamforming. However as $\gamma$ is increasing, we can find that the performance gap between two schemes become bigger.

Fig. 3. shows the convergence behavior of proposed power minimization technique. The vertical axis is average total transmit power and the horizontal axis means iteration number. In this simulation, we allocate initial values with $P_1=P_2=P_r=10$ dBW and $\gamma=3$ dB. We can see the $P_T$ of the proposed iterative algorithm converge to the minimum value averagely in 8 iteration with squared error less than $\|P_T(t)-P_T(t-1)\|^2 = 10^{-3}$. We can also check that this converged value $P_T$ is the same as the average total power corresponding to $\gamma=3$ dB in Fig. 2.

Fig. 4. shows the transition of received SNRs of transceiver1 and transceiver2 at each iteration until $P_T$ to converge. The simulation parameters $P_1$, $P_2$, $P_r$ and $\gamma$ are the same as those used in Fig. 3., for observe the variation of each SNRs during the iteration. In Fig. 4, we can find that at the converge point, the received SNRs of transceiver1 and transceiver2 satisfy equality with the minimum SNR for QoS.

In Figs. 3 - 4, we can see, when the $P_T$ at each iteration do not meeting the minimum SNR for QoS, our proposed iterative algorithm increase total transmit power gradually until the received SNRs of each transceiver meet the minimum SNR, as shown at iteration 3 to 8.
ZF criteria is formulated as follows: \( W \) ignoring noise. The optimized processing matrix vector received at each transceivers from relay, namely, \( \hat{s} \). From the numerical results we found that our proposed scheme shows better power minimization performance compared to equal power beamforming and eigen beamforming scheme. Moreover, we observed our iterative algorithm converged to the steady-state value in a reasonable number of iteration and at the converge point, received SNRs of each source were satisfied the required value in a reasonable number of iteration and at the converge point, received SNRs of each source were satisfied the required minimum SNR for QoS.

V. CONCLUSION

In this paper, we studied energy efficient two-way AF relay network equipped with multiple antennas at both transceivers and relay. To minimize the total transmit power under constraint of received SNRs of each source, we designed transmit beamformers and receive combiners through iterative algorithm with ZF based relay processing matrix. From the numerical results we found that our proposed scheme shows better power minimization performance compared to equal power beamforming and eigen beamforming scheme. Moreover, we observed our iterative algorithm converged to the steady-state value in a reasonable number of iteration and at the converge point, received SNRs of each source were satisfied the required minimum SNR for QoS.

APPENDIX A: ZF PROCESSING MATRIX

In this section, we introduce the relay processing matrix \( W \) based on ZF criteria when the transmit and receive combining vectors are given. The ZF processing matrix eliminates self-interference terms, \( \sqrt{P_1} d_1^H H_1^H W H_1 f_1 s_1 \) and \( \sqrt{P_2} d_2^H H_2^H W H_2 f_2 s_2 \) in (3) and (4), respectively, while ignoring noise. The optimized processing matrix \( W \) under the ZF criteria is formulated as follows:

\[
\min_{W} E \| s - P \hat{s}_f \|^2, \quad (20)
\]

where \( s = [s_1, s_2]^T \) is the signal vector transmitted from the transceivers and \( \hat{s}_f = [\hat{s}_{f,2}, \hat{s}_{f,1}]^T \) is the noise-free signal vector received at each transceivers from relay, namely,

\[
\hat{s}_f = D^H H^H W H F s,
\]

where \( F = \begin{bmatrix} f_1 & 0_{N_t} \\ 0_{N_t} & f_2 \end{bmatrix} \) and \( D = \begin{bmatrix} d_1 & 0_{N_t} \\ 0_{N_t} & d_2 \end{bmatrix}^H \) are beamforming matrix and receive combining matrix, respectively. \( H = [H_1 H_2] \) is a concatenated channel matrix and \( P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \) is a permutation matrix used to exchanging data of \( S1 \) and \( S2 \). Since (20) is convex with respect to \( W \), therefore, the optimal solution can be simply obtained as follows:

\[
\frac{\partial J}{\partial W} = HDD^H H^H W H F - HDPP^H H = 0.
\]

Hence, we obtain

\[
W = (D^H H^H)^{-1} P (HF)^{\dagger}.
\]

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